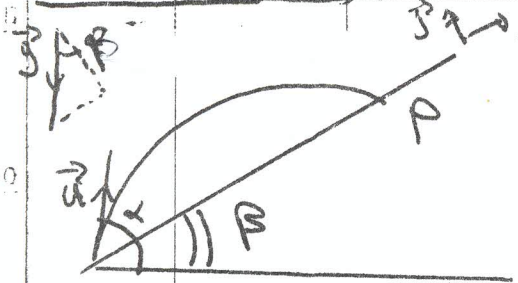
PROJECTILES

WATCH REDEFINITION!

1980, Q3 (Hons)



$\vec{u} = u\cos(\alpha-\beta)\,\vec{i} + u\sin(\alpha-\beta)\,\vec{j}$

$\vec{g} = -g\sin\beta\,\vec{i} - g\cos\beta\,\vec{j}$

* Told projectiles strikes plane at P, $\perp$ to plane *.

In general $\vec{v} = (u\cos(\alpha-\beta) - g\sin\beta\, t)\,\vec{i} + (u\sin(\alpha-\beta) - g\cos\beta\, t)\,\vec{j}$.

$\vec{r}(t) = \left(u\cos(\alpha-\beta)\,t - \frac{g}{2}\sin\beta\, t^2\right)\vec{i} + \left(u\sin(\alpha-\beta) - \frac{g}{2}\cos\beta\, t^2\right)\vec{j}$

Find t: when hits P (ie after time in flight) $(\vec{r}(t))\,\vec{j} = 0$

$\Rightarrow u\sin(\alpha-\beta)\,t - \frac{1}{2}g\cos\beta\, t^2 = 0$

$\Rightarrow t = 0$ OR $\boxed{t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}}$ (1) qed

Deduce $2\tan(\alpha-\beta)\tan\beta = 1$;

Projectile strike P $\perp$ to plane $\Rightarrow (\vec{v}(T))\,\vec{i}$

$(\vec{v}(t))\,\vec{i} = 0 \Rightarrow u\cos(\alpha-\beta) - g\sin\beta\, t = 0$

$\Rightarrow$

$\Rightarrow \boxed{T = \frac{u\cos(\alpha-\beta)}{g\sin\beta}}$ (2)

Equate (1), (2) $\Rightarrow$ $\frac{u\cos(\alpha-\beta)}{g\sin\beta} = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

$\Rightarrow 1 = 2\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}\frac{\sin\beta}{\cos\beta}$ $\Rightarrow \boxed{1 = 2\tan(\alpha-\beta)\tan\beta}$ qed

$(\alpha-\beta) = \frac{\pi}{4}$, $\Rightarrow \tan(\alpha-\beta) = 1$, & (2) $\Rightarrow$ $1 = 2\tan\beta(1) \Rightarrow \tan\beta = \frac{1}{2}$

$\tan(\alpha-\beta) = 1 \Rightarrow \sin(\alpha-\beta) = \cos(\alpha-\beta) = \frac{1}{\sqrt{2}}$

$\tan\beta = \frac{1}{2}$,

, $\Rightarrow \sin\beta = \frac{1}{\sqrt{5}}$, $\cos\beta = \frac{2}{\sqrt{5}}$

Range $= (\vec{r}(t))\,\vec{i} = u\cos(\alpha-\beta)\,t - \frac{g}{2}\sin\beta\, t^2$

$= u\left(\frac{1}{\sqrt{2}}\right)t - \frac{g}{2}\left(\frac{1}{\sqrt{5}}\right)t^2$

But in this particular case $t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta} = \frac{2u\frac{1}{\sqrt{2}}}{g\frac{2}{\sqrt{5}}} = \frac{u}{g}\frac{\sqrt{5}}{\sqrt{2}}$

$\therefore$ Range $= u\left(\frac{1}{\sqrt{2}}\right)\left[\frac{u}{g}\frac{\sqrt{5}}{\sqrt{2}}\right] - \frac{g}{2}\left(\frac{1}{\sqrt{5}}\right)\left(\frac{u}{g}\frac{\sqrt{5}}{\sqrt{2}}\right)^2 = \frac{u^2}{g}\frac{\sqrt{5}}{2} - \frac{u^2}{2g}\left(\frac{5}{2\sqrt{5}}\right)$

$= \frac{u^2}{g}\left[\frac{\sqrt{5}}{2} - \frac{\sqrt{5}}{4}\right] = \frac{u^2}{g}\frac{\sqrt{5}}{4}$ $\Rightarrow \boxed{\text{Range} = \frac{u^2}{g}\frac{\sqrt{5}}{4}}$