

HLC 43 April Q10

(a)  $(x^2+2) \frac{dy}{dx} = x(y+1)$   
 $y=2, x=1$  Find  $y$  when  $x=2$ .

$$(x^2+2) \frac{dy}{dx} = x(y+1) \quad // u = x^2+2$$

$$\Rightarrow \int \frac{1}{y+1} dy = \int \frac{x}{x^2+2} dx \quad \begin{matrix} \Rightarrow \frac{du}{dx} = 2x \\ \frac{1}{2} du = x dx \end{matrix}$$

$$\Rightarrow \ln(y+1) = \int \frac{1}{2} \frac{du}{u}$$

$$\Rightarrow \ln(y+1) = \frac{1}{2} \ln u + C$$

$$\Rightarrow \ln(y+1) = \frac{1}{2} \ln(x^2+2) + C$$

//  $x=1, y=2 \Rightarrow$

$$\ln(3) = \frac{1}{2} \ln(3) + C$$

$$\Rightarrow \ln 3 - \frac{1}{2} \ln 3 = C$$

$$\Rightarrow \frac{1}{2} \ln 3 = C$$

$$\therefore \ln(y+1) = \frac{1}{2} \ln(x^2+2) + \frac{1}{2} \ln 3$$

$$\Rightarrow \ln(y+1) = \frac{1}{2} \log(3(x^2+2))$$

$$\Rightarrow \ln(y+1) = \ln((3x^2+6)^{\frac{1}{2}})$$

$$\Rightarrow y+1 = \sqrt{3x^2+6}$$

$$\Rightarrow y = \sqrt{3x^2+6} - 1$$

$$x=2 \Rightarrow y = \sqrt{3(2)^2+6} - 1$$

$$y = \sqrt{18} - 1$$

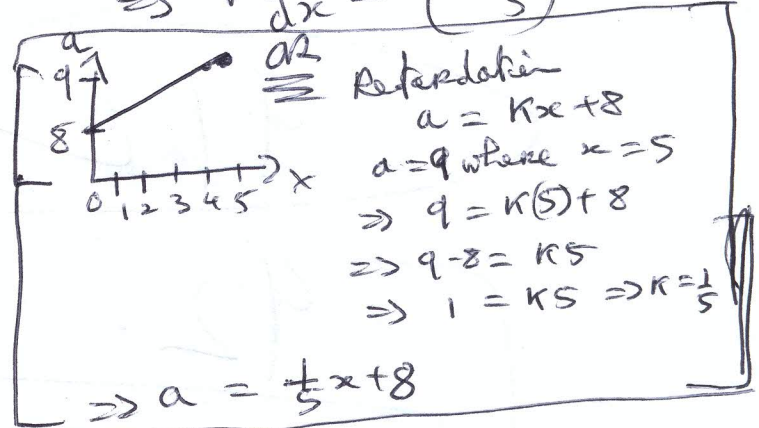
$$y = 3\sqrt{2} - 1 \text{ OR } 3.24$$

(b)  $x=0$   $x=5$   
 $t=0$   $t=?$   
 $v=20$   $v=?$   
 $a=8$   $a=-9$

(i) Retardation  $\propto x$   
 $\Rightarrow$  Retardation increases uniformly from 8 to 9 over 5 metres  $\Rightarrow$  Retardation is increasing at  $\frac{x}{5}$  m/s<sup>2</sup> every m.

$$\text{Retardation} = -\left(8 + \frac{x}{5}\right)$$

$$\Rightarrow v \frac{dv}{dx} = -\left(8 + \frac{x}{5}\right)$$



(ii) Find  $x$  when  $v=0$

$$\begin{matrix} v=20 & v=0 \\ x=0 & x=x \end{matrix}$$

$$v \frac{dv}{dx} = -\left(8 + \frac{x}{5}\right)$$

$$\Rightarrow \int_{20}^0 v dv = \int_0^x -\left(8 + \frac{x}{5}\right) dx$$

$$\Rightarrow \frac{v^2}{2} \Big|_{20}^0 = -8x - \frac{x^2}{10} \Big|_0^x$$

$$\frac{0^2}{2} - \frac{(20)^2}{2} = \left(-8x - \frac{x^2}{10}\right) - (0)$$

$$\Rightarrow -200 = -8x - \frac{x^2}{10}$$

$$\Rightarrow x^2 + 80x - 2000 = 0$$

$$\Rightarrow (x-20)(x+100) = 0$$

$$\Rightarrow x-20=0 \text{ OR } x+100=0$$

$$\Rightarrow x=20 \text{ OR } (x=-100)$$

$$\Rightarrow \boxed{x=20 \text{ metres}}$$