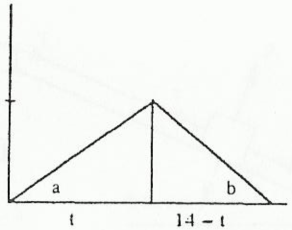
1(a)

$\frac{d_1}{d_2} = \frac{t_2}{t_1}$



$\tan a = \frac{v}{t} \Rightarrow v = 0.6t$

$\tan b = \frac{v}{14 - t} \Rightarrow v = 0.8(14 - t)$

$0.6t = 0.8(14 - t)$

$\Rightarrow t = 8$

$\Rightarrow v = 4.8$

distance $= 0.5(14)(4.8) = 33.6$ m.

(b) (i) upwards $98 - mg = mf$

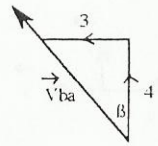
downwards $mg - 68.6 = 2mf$

$\Rightarrow mg = 88.2$ or $m = 9$ kg

(ii) $98 - 88.2 = 9f$

$\Rightarrow 2f = \frac{2g}{9}$

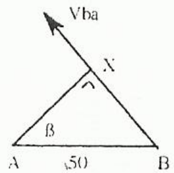
2(a) (i) $\vec{V}ba = \vec{V}b - \vec{V}a$



$|\vec{V}ba| = 5$

$\tan ß = 3/4$

(ii)



After 10 seconds B is at p and A is (80 − 3x10 =) 50 m from p. They are nearest together when B reaches X.

time $= \frac{BX}{Vba} = \frac{50 \sin ß}{5} = 6$ seconds

total time = 16 seconds

After 16 s A is (80 − 3x16 =) 32 m from p

After 16 s B is (40 − 4x16 =) −24 m from p

i.e. 24 m past p

(b) (i) $\vec{V}ba = \vec{V}b - \vec{V}a$

$= (4 - qt)\vec{j} - (3 + 0.1t)\vec{i}$

(ii) Find time it takes A to reach p

$3t + 0.5(0.1)t^2 = 80$ $\quad .05t^2 + 3t - 80 = 0$

$\Rightarrow t = 20$

As they arrive together, the time for B to reach p is 20 s

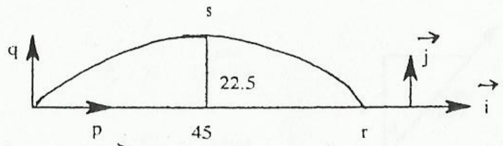
$4t - 0.5qt^2 = 40$

$4(20) - 0.5q(400) = 40$

$\Rightarrow q = 0.2$

3(a) (i)

1994



$\vec{r} = (p.t)\vec{i} + (q.t - 0.5gt^2)\vec{j}$

$\vec{v} = (p)\vec{i} + (q - gt)\vec{j}$

At r $\quad r_{\vec{j}} = 0 \quad \Rightarrow \quad t = \frac{2q}{g}$

$\Rightarrow$ time to reach s $= \frac{q}{g}$

At s $\quad r_{\vec{j}} = 22.5 \quad \Rightarrow \quad q.t - 0.5gt^2 = 22.5$

$\frac{q^2}{g} - \frac{q^2}{2g} = 22.5$

$q^2 = 22.5(2g)$

$q = 21$

At r $\quad r_{\vec{i}} = 45 \quad \Rightarrow \quad p.t = 45$

$p.\frac{2q}{g} = 45$

$\Rightarrow \quad p = 10.5$

(ii) time of flight $= \frac{2q}{g} = \frac{42}{g}$

second player can be up to $\left(7 \times \frac{42}{g} = \right)$ 30 metres from r

(b) (i) $\vec{r} = ut\,\vec{i} + (0 - 0.5gt^2)\,\vec{j}$

$v = u\,\vec{i} - gt\,\vec{j}$

$r_{\vec{j}} = -0.3 \quad \Rightarrow \quad 0.5gt^2 = 0.3$

$t = \frac{\sqrt{3}}{7}$ or 0.247

Range $= 3$

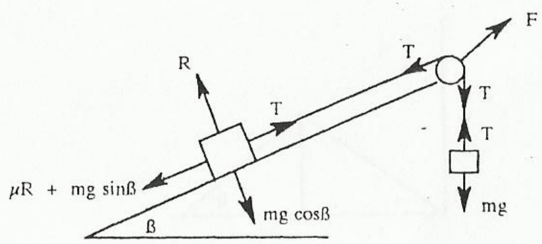
$ut = 3$

$u = 7\sqrt{3}$ or 12.12

(ii) $v = 7\sqrt{3}\,\vec{i} - 1.4\sqrt{3}\,\vec{j}$

$|v| = 12.36$

4 (i)



(ii) $T - \mu R - mg\sin\beta = mf$

$mg - T = mf$ ✓

$\Rightarrow \quad T - \mu R - mg\sin\beta = mg - T$

$2T = \mu(mg\cos\beta) + mg\sin\beta + mg$

$2T = \frac{4}{13}mg + \frac{5}{13}mg + mg$

$T = \frac{11}{13}mg$

(iii) Stage 1 - first two seconds TWO STAGES.

From equation (2) $\quad f = \frac{2g}{13}$

$s = ut + 0.5ft^2$

$= 0 + \frac{1}{2}\frac{2g}{13}(4)$

$= \frac{4g}{13}$

$v = 0 + \frac{2g}{13}(2) = \frac{4g}{13}$

Stage 2 $\quad f = -\mu g\cos\beta - g\sin\beta = -\frac{9g}{13}$

$v^2 = u^2 + 2fs$

$0 = \frac{16g^2}{169} - \frac{18gs}{13}$

$s = \frac{8g}{117}$

Total distance $= \frac{36g}{117} + \frac{8g}{117} = \frac{44g}{117}$

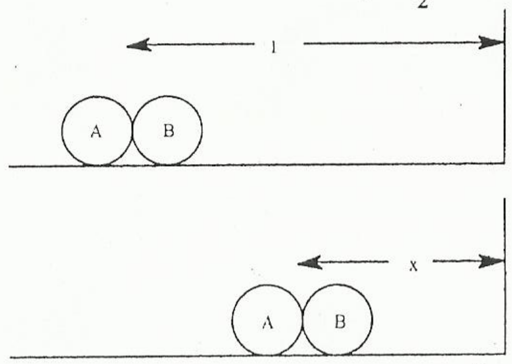
1994

5 PCM $\quad mu + 0 = mv_1 + mv_2 \Rightarrow \quad v_1 + v_2 = u$

NEL $\quad v_1 - v_2 = -e(u - 0) \Rightarrow \quad v_1 - v_2 = -eu$

$$v_1 = \frac{u}{2}(1 - e)$$

$$v_2 = \frac{u}{2}(1 + e)$$



B strikes the wall with velocity $\frac{u}{2}(1 + e)$ and rebounds with velocity $\frac{eu}{2}(1 + e)$

$$\frac{l - x}{\frac{u}{2}(1 - e)} = \frac{l}{\frac{u}{2}(1 + e)} + \frac{x}{\frac{eu}{2}(1 + e)}$$

$$x = \frac{2e^2}{1 + e^2}$$

OR

Time for B to reach the wall $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{l}{\frac{u}{2}(1 + e)} = \dfrac{2}{u(1 + e)}$

In this time A travels $\frac{u}{2}(1 - e) \cdot \frac{2}{u(1 + e)} = \frac{1 - e}{1 + e}$

and is now $\quad 1 - \frac{1 - e}{1 + e} = \frac{2e}{1 + e}$ from the wall

B's rebound velocity is $\frac{eu}{2}(1 + e)$

$$\frac{x}{\frac{eu}{2}(1 + e)} = \frac{\frac{2e}{1 + e} - x}{\frac{u}{2}(1 - e)}$$

$$x = \frac{2e^2}{1 + e^2}$$

IGNORE

6 (i) amplitude $= (1.5 - 0.9) \div 2 = 0.3$ metres

1994

Period $= \frac{2\pi}{\omega} = \frac{\pi}{6} \quad \Rightarrow \quad \omega = 12$ rad/s

max speed $= \omega a = 12 \times 0.3 = 3.6$ m/s

(ii) Maximum force that the glue has to exert is at the **highest point**, when

maximum acceleration $= \omega^2 a$

$= 144 \times 0.3$

$= 43.2$ (downwards)

Let F be the force that the glue exerts on Q

Force = mass x acceleration

$mg + F = mf$

$0.2(9.8) + F = 0.2(43.2)$

$F = 6.68$ N

(iii) In the absence of glue, Q will leave the pan when $R = 0$

$mg - R = mf$

$mg - 0 = m\omega^2 x$

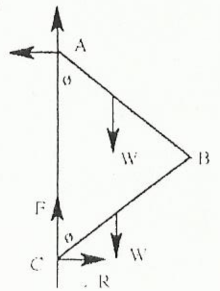
$9.8 = 144x$

$x = 0.068$ m

$\Rightarrow$ length of spring $= 1.2 - 0.068 = 1.132$ m

1994

7 (i)



Moments about A for system

$$W(0.5\ell\sin\phi) + W(0.5\ell\sin\phi) = R(2\ell\cos\phi)$$

$$\Rightarrow \quad R = \frac{W\tan\phi}{2}$$

Moments about B for rod BC

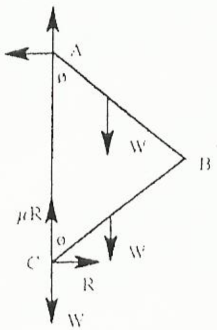
$$W(0.5\ell\sin\phi) + R(\ell\cos\phi) = F(\ell\sin\phi)$$

$$W\tan\phi + 2R = 2F\tan\phi$$

$$W\tan\phi + W\tan\phi = 2F\tan\phi$$

$$\Rightarrow \quad F = W$$

(ii)



Moments about B for rod BC

$$W(0.5\ell\sin\phi) + W(\ell\sin\phi) + R(\ell\cos\phi) = \mu R(\ell\sin\phi)$$

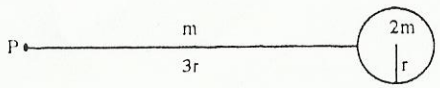
$$1.5W\tan\phi + R = \mu R\tan\phi$$

$$1.5(2R) + R = \mu R\tan\phi$$

$$\Rightarrow \quad \mu = \frac{4}{\tan\phi}$$

8 (i)

~~1993~~



1994

$$I \text{ about p} = \frac{4}{3}m\left(\frac{3r}{2}\right)^2 + \frac{1}{2}(2m)r^2 + 2m(4r)^2$$

$$= 3mr^2 + mr^2 + 32mr^2$$

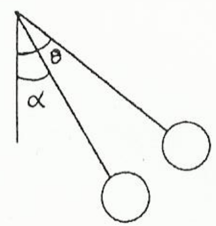
$$\Rightarrow \quad I = 36mr^2$$

$$Mh = m(1.5r) + 2m(4r)$$

$$\Rightarrow \quad Mh = \frac{19}{2}mr$$

$$\text{Period} \quad T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{72r}{19g}}$$

(ii)



Gain in kinetic energy = Loss in potential energy

$$\frac{1}{2}I\omega^2 = mg(1.5r)(\cos\alpha - \cos\theta) + 2mg(4r)(\cos\alpha - \cos\theta)$$

$$= \frac{19}{2}mgr(\cos\alpha - \cos\theta)$$

$$\omega^2 = \frac{19mgr}{36mr^2}(\cos\alpha - \cos\theta)$$

$$\Rightarrow \quad \omega^2 = \frac{19g}{36r}(\cos\alpha - \cos\theta)$$

1994

9(a) (i) mass of sea-water displaced $= 200$ tonnes

volume of water displaced $= 200000 \div 1030 = 194.17\ m^3$

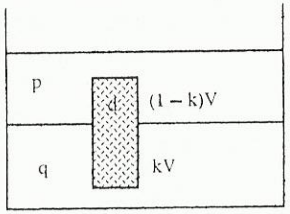
(ii) This volume of fresh water has a mass $= 194.17$ tonnes

$\Rightarrow$ Submarine must reduce its mass to 194.17 tonnes

by pumping out 5.83 tonnes of water from ballast tanks

(b) Let k be the fraction of the volume of the solid immersed in the lower liquid (of density q).

$\Rightarrow$ $(1 - k)$ is the fraction in the upper liquid (of density p)



$$B_p + B_q = W$$

$$\frac{(1-k)Vdg\frac{p}{1000}}{d/1000} + \frac{kVdg\frac{q}{1000}}{d/1000} = Vdg$$

$$\Rightarrow \quad (1-k)p + kq = d$$

$$k = \frac{d-p}{q-p}$$

1994

10(a)

$$\int \frac{dy}{y} = \int \left(\frac{1-x}{1+x}\right) dx \quad \Leftarrow \text{difficult}$$

$$= \int \left(-1 + \frac{2}{1+x}\right) dx$$

$$\ln y = -x + 2\ln(1+x) + C$$

$$\ln 1 = -0 + 2\ln(1+0) + C$$

$$C = 0$$

$$\Rightarrow \quad \ln y = 2\ln(1+x) - x$$

$$y = e^{2\ln(1+x) - x} \quad \text{or} \quad y = (1+x)^2 e^{-x}$$

(b) (i)

Rarely done correctly

Power = Tractive force x velocity

$75000 = T\,v$

Force = mass x acceleration

$T - 1500 = 1000\,f$

$\frac{75000}{v} - 1500 = 1000\,f$

$$\Rightarrow \quad f = \frac{75}{v} - 1.5 = \frac{150 - 3v}{2v}$$

(ii)

$$\frac{dv}{dt} = \frac{3(50 - v)}{2v}$$

$$\int_0^{25} \frac{v\,dv}{50 - v} = 1.5\int_0^t dt$$

$$\int_0^{25} \left(-1 + \frac{50}{50 - v}\right) dv = 1.5\int_0^t dt$$

$$\Big[-v - 50\ln(50 - v)\Big]_0^{25} = 1.5\,t$$

$$\Rightarrow \quad t = 6.44 \text{ seconds}$$

John Maher
Dublin Branch