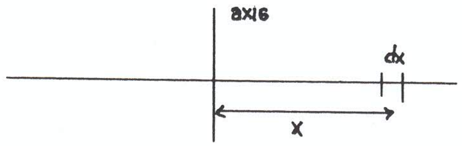
91

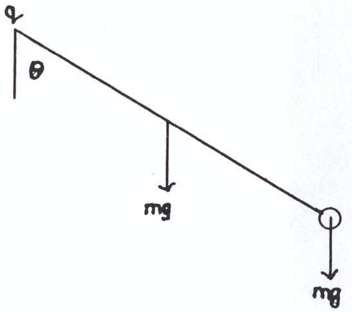
Q 8(a)



Let m = mass per unit length
mass of element = $m\,dx$
moment of inertia of element = $(m\,dx)\,x^2$

$$I = m\int_{-\ell}^{\ell} x^2 . dx$$
$$= \tfrac{2}{3} m\, \ell^3$$
$$= \tfrac{1}{3} M\, \ell^2 \qquad \text{where } M = 2m\ell \text{ is the mass of the rod}$$

(b) (i)



$$I = \tfrac{4}{3} m(0.44)^2 + \tfrac{1}{2} m(.12)^2 + m(1)^2 = 1.265\ m$$

$$Mh = m(0.44) + m(1) = 1.44\ m$$

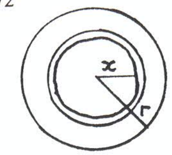
$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{1.265}{1.44g}} = 1.88\ s$$

(ii) length of simple pendulum = I/Mh

$$= 1.265\ m / 1.44\ m$$
$$= 0.88\ m$$

89

Q. 8 (a)



Let m = mass per unit area

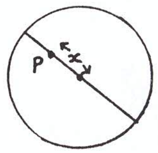
mass of element = $m(2\pi x.dx)$

moment of inertia of element = $m(2\pi x.dx)x^2$

$$I = 2\pi m \int_0^r x^3 . dx$$
$$= 2\pi m \left[\frac{x^4}{4}\right]_0^r$$
$$= \tfrac{1}{2} M r^2$$

where $M = m\pi r^2$ is the mass of the lamina

(b)



$$I_P = \tfrac{1}{2}Mr^2 + Mx^2$$

$$T = 2\pi\sqrt{\frac{I}{Mgh}}$$

$$= 2\pi\sqrt{\frac{\tfrac{1}{2}Mr^2 + Mx^2}{Mgx}} = 2\pi\sqrt{\frac{r^2 + 2x^2}{2gx}}$$

$$\frac{dT}{dx} = 2\pi . \frac{1}{2}\sqrt{\frac{2gx}{r^2 + 2x^2}}\left\{\frac{2gx(4x) - (r^2 + 2x^2)2g}{(2gx)^2}\right\} = 0$$

$$\Rightarrow \quad 8gx^2 = 2gr^2 + 4gx^2$$

$$x = \frac{r}{\sqrt{2}}$$