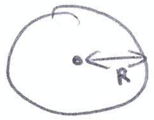
1981 H :

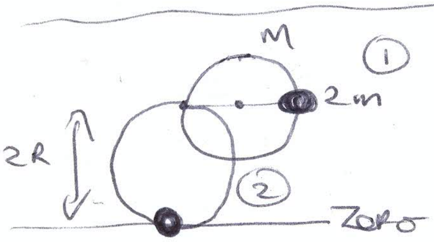


density ρ.

mass disk M	mass strip Δm
Area disk πR^2	Area disk $(2\pi x)\Delta x$

$$\Delta I = \Delta m\, x^2$$
$$= (\rho 2\pi x\, \Delta x)\, x^2$$
$$\int dI = \int_0^R \rho 2\pi x^3 dx$$
$$= \rho 2\pi \int x^3 dx$$
$$= \rho 2\pi \frac{x^4}{4}$$
$$= \rho 2\pi \frac{R^4}{4}$$
$$= (\rho \pi R^2)\frac{R^2}{2}$$
$$I = \frac{1}{2} m R^2$$

$$I_p = I_c + m(pc)^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 \text{ for disk.}$$



$$I_{p(syst)} = I_{p\,\bigcirc} + I_{p\,\bullet}$$
$$I_{p\,\bigcirc} = \frac{3}{2}mR^2$$
$$I_{p\,\bullet} = [2m](2R)^2 = 8mR^2$$
$$\Rightarrow I_{p(syst)} = \frac{19}{2}mR^2.$$

Position ①

$$\varepsilon_1 = mgR + (2m)g(2R) + \frac{1}{2}I(0)^2$$
$$= 6mgR$$

Position ② :

$$\varepsilon_2 = MgR + (2m)g(0) + \frac{1}{2}I\omega^2$$
$$= mgR + \frac{1}{2}\left(\frac{19}{2}mR^2\right)\omega^2$$

P C E $\Rightarrow$ $\varepsilon_1 = \varepsilon_2$

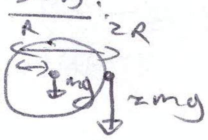
$$\Rightarrow 6mgR = mgR + \frac{19}{4}mR^2\omega^2$$
$$\Rightarrow \frac{20g}{19R} = \omega^2 \Rightarrow \boxed{\omega = \sqrt{\frac{20g}{19R}}}$$

Find T : $I = \frac{19}{2}mR^2$, Total $m = m + 2m = 3M$. ,

$T = 2\pi\sqrt{\frac{I}{Mgh}}$

h :

Bits.



whole

$$\Rightarrow mgR + 2mg(2R) = 3mg\,h$$
$$\Rightarrow \frac{5R}{3} = h$$

$$\therefore T = 2\pi\sqrt{\frac{\frac{19}{2}mR^2}{3mg\left(\frac{5R}{3}\right)}} = 2\pi\sqrt{\frac{19R}{10g}}$$

$$T_{simple} = 2\pi\sqrt{\frac{L}{g}} \Rightarrow 2\pi\sqrt{\frac{19R}{10g}} = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \boxed{L = \frac{19R}{10}} \text{ qed.}$$