

Eq 1 (1993)

(a) A particle of mass m moves with simple harmonic motion under the action of a variable force. If the maximum value of the force is $\frac{7m}{16}$ and the amplitude of the motion is 4 m calculate

(i) the period of the oscillation.

(ii) the speed of the particle at a time $\frac{2\pi}{\sqrt{7}}$ seconds after passing through the centre of oscillation.

(a) NII \Rightarrow Net $F = ma$

SHM \Rightarrow Force = $m\omega^2 x$ about $x=0$

Told $\Rightarrow \frac{7}{16}m = m\omega^2 A$ [max force at extreme]

$\Rightarrow \frac{7}{16}m = m\omega^2 4$

$\Rightarrow \frac{7}{64} = \omega^2 \Rightarrow \omega = \frac{\sqrt{7}}{8} \text{ Rad/sec}$

(i) $T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\sqrt{7}}{8}} = \frac{16\pi}{\sqrt{7}}$ seconds

(ii) Find v

$x=0$ $x=?$

$t=0$ $t = \frac{2\pi}{\omega}$

sp $\frac{1}{\sqrt{7}}$

$v=?$

First x :

$x = A \sin \omega t$

$x = 4 \sin \left(\frac{\sqrt{7}}{8} \cdot \frac{2\pi}{\sqrt{7}} \right)$

$x = 4 \sin \left(\frac{\pi}{4} \right)$

$x = 4 \cdot \frac{1}{\sqrt{2}}$

$x = 2\sqrt{2}$

Now v :

$v^2 = \omega^2 (A^2 - x^2)$

$v^2 = \left(\frac{\sqrt{7}}{8} \right)^2 (4^2 - (2\sqrt{2})^2)$

$v^2 = \frac{7}{64} (16 - 8)$

$v^2 = \frac{7}{64} (8)$

$v^2 = \frac{7}{8} \Rightarrow v = \sqrt{\frac{7}{8}} = 0.94 \text{ sec}$

OR (ii) $x = A \sin \omega t$

$\Rightarrow v = \frac{dx}{dt} = +A\omega \cos \omega t$

$\Rightarrow v = 4 \left(\frac{\sqrt{7}}{8} \right) \cos \left[\frac{\sqrt{7}}{8} \left(\frac{2\pi}{\sqrt{7}} \right) \right]$

$v = \frac{\sqrt{7}}{2} \cos \left(\frac{\pi}{4} \right)$

$v = \frac{\sqrt{7}}{2} \cdot \frac{1}{\sqrt{2}} = \sqrt{\frac{7}{8}}$ as before

(b) A light elastic string, of elastic constant $\frac{48mg}{l}$ and natural length l has one end attached to a fixed point. Two particles of masses $3m$ and $2m$ are attached to the other end and the system hangs in equilibrium. If the $2m$ mass falls off

(i) prove that the $3m$ mass will move with simple harmonic motion of period

(ii) find the amplitude of the motion.

(i) Let equil be y below ceiling.

$a=0$

↓ $l_0 = l$ y forces Hooke

$k = \frac{48mg}{l}$ $\uparrow s$ $\downarrow 3mg$ $|s| = \frac{48mg}{l} (y-l)$

NII ($a=0$) $\Rightarrow -|s| + 3mg = 0$

$\Rightarrow -\frac{48mg}{l} (y-l) + 3mg = 0$

$\Rightarrow -\frac{48y}{l} + 48 + 3 = 0$

$\Rightarrow 48y = 51l$

$\Rightarrow y = \frac{17l}{16}$

Equil is $\frac{17l}{16}$ below ceiling.

Examine forces at $\frac{17l}{16} + x$ below ceiling.

↓ l $\frac{17l}{16} + x$ forces Hooke $s' = k(x+l)$

$\uparrow s'$ $\downarrow 3mg$ $s' = \frac{48mg}{l} \left(\frac{l}{16} + x \right)$

NII: $\Sigma F = ma$

$\Rightarrow -\frac{48mg}{l} \left(\frac{l}{16} + x \right) + 3mg = 3ma$

$\Rightarrow -3mg - \frac{48mg}{l} x + 3mg = 3ma$

$\Rightarrow -\frac{16g}{l} x = a$

SHM with $\omega = 4\sqrt{\frac{g}{l}}$

$\therefore T = \frac{2\pi}{\omega} = \frac{\pi}{2} \sqrt{\frac{l}{g}}$ qed.

(ii) Find A Find where particle started from

ie where $2m$ mass drops off and where accel = $\omega^2 A = \left(4\sqrt{\frac{g}{l}} \right)^2 A$

↓ $\frac{17l}{16}$ forces Hooke

$\uparrow T''$ $\downarrow 5m$ $T'' = k \left(\frac{17l}{16} + A - l \right)$

$T'' = \frac{48mg}{l} \left[\frac{l}{16} + A \right]$

NII $\Rightarrow \Sigma F = ma$

$\Rightarrow -\frac{48mg}{l} \left[\frac{l}{16} + A \right] = 5m \left[\left(4\sqrt{\frac{g}{l}} \right)^2 A \right]$

$\Rightarrow A = \frac{l}{24}$