

(a)(i) $x = 5 \cos \omega t + 12 \sin \omega t$.

$\frac{dx}{dt} = -5\omega \sin \omega t + 12\omega \cos \omega t$

$\frac{d^2x}{dt^2} = -5\omega^2 \cos \omega t - 12\omega^2 \sin \omega t$.

$\frac{d^2x}{dt^2} = -\omega^2 [5 \cos \omega t + 12 \sin \omega t]$

$\frac{d^2x}{dt^2} = -\omega^2 x$.

(ii) Amplitude is the x maximum which occurs where

$v = \left(\frac{dx}{dt}\right) = 0 \therefore -5\omega \sin \omega t + 12\omega \cos \omega t = 0$.

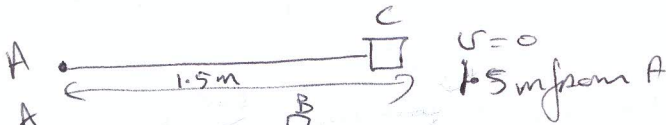
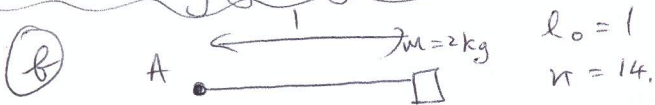
$\Rightarrow 5 \sin \omega t = 12 \cos \omega t$

$\Rightarrow \tan \omega t = \frac{12}{5}$

$\Rightarrow \frac{\omega t}{5} = \frac{12}{13} \Rightarrow \omega t = \frac{12}{13}$ and $\cos \omega t = \frac{5}{13}$

$\therefore A = x = 5 \left[\frac{5}{13}\right] + 12 \left[\frac{12}{13}\right] = \frac{169}{13} = 13$.

[OR if you guessed 5, 12, \Rightarrow 13 then you got the marks!]]



(i) Find Equil position: A

Accel = 0

Forces =

N II:

$\sum F = ma \Rightarrow - (14y - 14) = 2(0)$

Hookes $\Rightarrow |S| = 14(y - 1) = 14y - 14$

$\Rightarrow y = 1$

Example Typical position x m from B [where $x = 0$]

Accel = a

Forces

N II:

$\sum F = ma \Rightarrow -14x = 2(a) \Rightarrow a = -7x$

Hookes =

$|S| = 14(1 + x - 1) = 14x$.

(ii) Time from C (Extreme) to B (mean)

$x = A \cos \omega t$

$A_{BB} x = 0 \Rightarrow$

$0 = 5 \cos \omega t \Rightarrow \omega t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$

SHM about pt B with $\omega = \sqrt{7}$.
 $v = 5$ at 1.5 from A \Rightarrow Amp = 0.5.

$\omega t = \frac{\pi}{4} = \frac{1}{4} \left(\frac{2\pi}{\omega}\right)$
 $= \frac{1}{4} \frac{2\pi}{\sqrt{7}} = \frac{\pi}{2\sqrt{7}}$

(ii) $\frac{4}{\sqrt{7}}$ sec with constant speed \Rightarrow Find time to travel

Time = $\frac{2|BA|}{\text{speed at B}} = \frac{2}{\text{speed at B}}$

At pt B,

$x = 0, v^2 = \omega^2(A^2 - x^2) \Rightarrow v^2 = 7(0.5)^2 - 0 = 7(0.5)^2 \Rightarrow v = \sqrt{7}(0.5)$

\therefore Time = $\frac{2}{\sqrt{7}(0.5)} = \frac{4}{\sqrt{7}}$ sec q.e.d.

