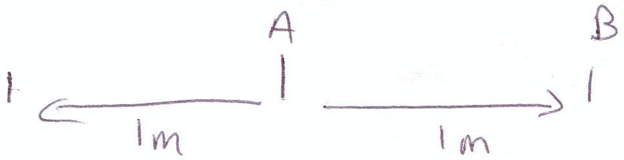
(ii) Time to travel C to B = Time to travel C (Extreme) to B (mean)

$$\therefore = \frac{1}{4} T$$
$$= \frac{1}{4}\left(\frac{2\pi}{\omega}\right)$$
$$= \frac{1}{4}\left(\frac{2\pi}{\sqrt{7}}\right)$$
$$= \frac{\pi}{2\sqrt{7}} \text{ seconds.}$$

(iii) Particle will travel at constant speed for as long as the string is slack. = time to travel from B to A and then 1m (natural length of string) beyond A.



$$\text{Time} = \frac{2\,|BA|}{\text{Speed at } B} = \frac{2(1)}{v_B}.$$

At point B: $x = 0 \Rightarrow v^2 = \omega^2(A^2 - x^2)$

$$\Rightarrow v^2 = (\sqrt{7})^2((.5)^2 - 0^2)$$
$$\Rightarrow v^2 = \frac{7}{4}$$
$$\Rightarrow v = \sqrt{\frac{7}{4}}.$$

$$\therefore \text{Time} = \frac{2}{\left(\frac{\sqrt{7}}{2}\right)} = \frac{4}{\sqrt{7}} \text{ seconds as required.}$$

1942. (a) If the displacement of a moving particle at any time t is given by the equation

$$x = 5\cos\omega t + 12\sin\omega t$$

(i) show that the motion is a simple harmonic motion.
(ii) calculate the amplitude of the motion.

(i) $x = 5\cos\omega t + 12\sin\omega t$

$$\frac{dx}{dt} = -5\omega\sin\omega t + 12\omega\cos\omega t$$

$$\frac{d^2x}{dt^2} = -5\omega^2\cos\omega t \quad 12\omega^2\sin\omega t.$$

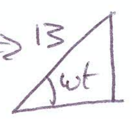
$$\frac{d^2x}{dt^2} = -\omega^2[5\cos\omega t + 12\sin\omega t]$$

$$\frac{d^2x}{dt^2} = -\omega^2 x, \text{ as required}$$

(ii) To find A

$A = x_{max}$ occurs where $v = 0$.

$$\Rightarrow \frac{dx}{dt} = 0$$
$$\Rightarrow -5\omega\sin\omega t + 12\omega\cos\omega t = 0$$
$$\Rightarrow 5\sin\omega t = 12\cos\omega t.$$
$$\Rightarrow \tan\omega t = \frac{12}{5}.$$



$\Rightarrow \cos\omega t = \frac{5}{13}$ and $\sin\omega t = \frac{12}{13}$

$$\therefore A = x_{max} = 5\left[\frac{5}{13}\right] + 12\left[\frac{12}{13}\right] = 13$$

[or guess 5, 12 ⇒ 13 !]