Eg 3 (1990)

6. (a) A particle starts from rest, and moves with simple harmonic motion of period $6n$ seconds. Show that the particle moves from the position of maximum velocity to the position in which the velocity is half the maximum in n seconds.

(b) The depth of water in a harbour is assumed to rise and fall with time in simple harmonic motion. On a certain day the low tide had a height of 13 m at 12.58 p.m. and the following high tide had a height of 18 m at 6.58 p.m.

If a ship requires a depth of 16·5 m of water before it can leave the harbour, find the latest time on that day that the ship can leave the harbour.

(a) $\left.\begin{array}{l} T = 6n \\ T = \frac{2\pi}{\omega}\end{array}\right] \Rightarrow \frac{2\pi}{\omega} = 6n$

$\Rightarrow \omega = \frac{2\pi}{6n}$

$\Rightarrow \boxed{\omega = \frac{\pi}{3n}}$

First find v_{max} : (occurs at $x=0$)

$v_{max} = \omega A$ [From $v^2 = \omega^2(A^2 - x^2)$]

$\frac{v_{max}}{2} = \frac{\omega A}{2}$

Find the position of $\frac{v_{max}}{2}$

$v^2 = \omega^2(A^2-x^2)$

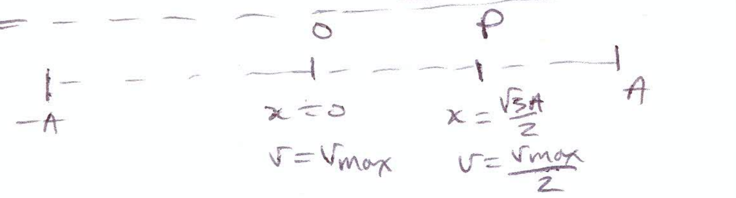
$\Rightarrow \left(\frac{\omega A}{2}\right)^2 = \omega^2(A^2-x^2)$

$\Rightarrow \frac{\omega^2 A^2}{4} = \omega^2(A^2-x^2)$

$\Rightarrow \frac{A^2}{4} = A^2 - x^2$

$\Rightarrow x^2 = \frac{3A^2}{4}$

$\Rightarrow x = \pm\frac{\sqrt{3}A}{2}$



Find t_{op} :

$x = A\sin\omega t$ [From $x=0$]

$\therefore \frac{\sqrt{3}A}{2} = A\sin\left(\frac{\pi}{3n}t\right)$

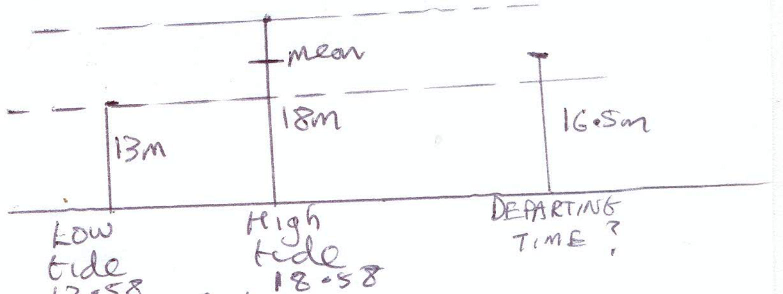
$\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{3n}t\right)$

$\sin^{-1} \Rightarrow \frac{\pi}{3} = \frac{\pi}{3n}t$

$\Rightarrow \frac{\pi}{3}\left(\frac{3n}{\pi}\right) = t$

$\Rightarrow \boxed{n \text{ seconds} = t}$ qed.

(b) Tide Rises and falls with SHM.



Must calculate: Mean position, ω, A.

Find A: High − Low = 18 − 13 = 5

$\Rightarrow A = \frac{5}{2} = 2{\cdot}5$

$\therefore$ Mean position is 13 + 2.5 = 15.5m

Find ω:

Time between high and low = 18·58 − 12·58 = 6 hours

$\Rightarrow \frac{1}{2}$ periodic time = 6 hours

$\Rightarrow$ periodic time T = 12 hours

[Time low to high to low]

$\therefore T = \frac{2\pi}{\omega} \Rightarrow 12 = \frac{2\pi}{\omega}$

$\Rightarrow \boxed{\omega = \frac{\pi}{6}}$

Latest time to reach 16·5m = Time from high tide (18:58) + time to travel from high extreme to 16·5.

Depth 16·5 $\Rightarrow x = 16{\cdot}5 - 15{\cdot}5 = 1$ (mean)

$\therefore x = A\cos\omega t$

$\Rightarrow 1 = 2{\cdot}5\cos\left(\frac{\pi}{6}t\right)$

$\Rightarrow 2{\cdot}215$ hours $= t$

$\Rightarrow$ 2 hrs 13 mins = t

$\therefore$ Latest time is 18:58 + 2:13 = 21:11