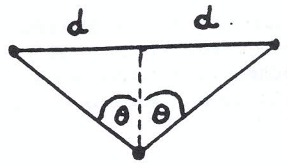
APPENDIX 3: HOOKES LAW EXAMPLE

1981

8. (a) A heavy particle is hung from two points on the same horizontal line and a distance $2d$ apart by means of two light, elastic strings of natural length l_1, l_2 and elastic constants k_1, k_2 respectively.

In the equilibrium position the two strings make equal angles θ with the vertical.

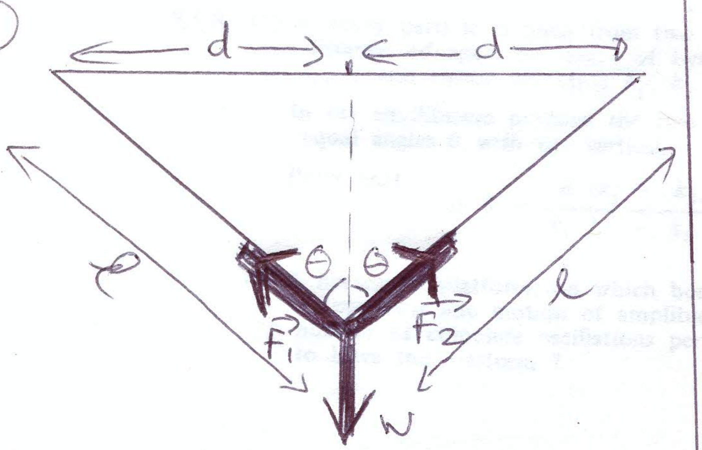


Prove that

$$\sin\theta = \frac{d\,(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$$

(b) A horizontal platform, on which bodies are resting, oscillates vertically with simple harmonic motion of amplitude 0·2 m. What is the maximum integral number of complete oscillations per minute it can make, if the bodies are not to leave the platform ?

(a)



$\vec{F_1} = -F_1 \sin\theta\,\vec{i} + F_1 \cos\theta\,\vec{j}$

$\vec{F_2} = F_2 \sin\theta\,\vec{i} + F_2 \cos\theta\,\vec{j}$.

$\vec{W} = -mg\vec{j}$

Equilibrium in the $\vec{i}$ direction

$\Rightarrow F_1 \sin\theta = F_2 \sin\theta$

$\Rightarrow \boxed{F_1 = F_2}$

Hooke Law: $|F_1| = k_1(l - l_1)$

$|F_2| = k_2(l - l_2)$

$\therefore k_1(l - l_1) = k_2(l - l_2)$

$\Rightarrow k_1 l - k_1 l_1 = k_2 l - k_2 l_2$

$\Rightarrow k_1 l - k_2 l = k_1 l_1 - k_2 l_2$

$\Rightarrow (k_1 - k_2) l = k_1 l_1 - k_2 l_2$

$\Rightarrow l = \dfrac{k_1 l_1 - k_2 l_2}{k_1 - k_2}$

But geometry $\Rightarrow$

$\sin\theta = \dfrac{d}{l}$

$\Rightarrow \sin\theta = \dfrac{d}{\left(\dfrac{k_1 l_1 - k_2 l_2}{k_1 - k_2}\right)}$

$\Rightarrow \sin\theta = \dfrac{d(k_1 - k_2)}{k_1 l_1 - k_2 l_2}$ q.e.d.

(b) Platform performing SHM

$A = .2$

Won't leave platform

$\Leftrightarrow$ max accel $\leq g$

SHM |max accel| $= \omega^2 A$.

$\therefore \omega^2(.2) \leq 9.8$

$\Rightarrow \omega^2 \leq 49$

$\Rightarrow \omega \leq 7$.

Take ω at its max value 7.

$\therefore T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{7} = .8977$ secs.

1 osc takes .8977 seconds

No of oscillations per minute $= \dfrac{60}{.8977}$

$= 66$ to nearest integer