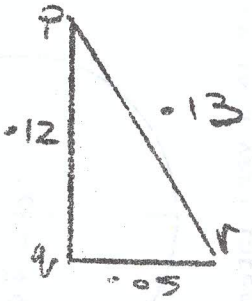
CIRCULAR MOTION (M) 1982 Q3.

(ii) Geometry -



$|PR| + |RQ| = .18$

Let $|PR| = x$

$|RQ| = .18 - x$

Pythagoras $\Rightarrow$

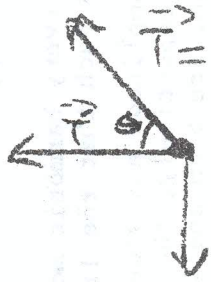
$(.12)^2 + (.18-x)^2 = x^2$

$\Rightarrow \quad x = .13$

$\Rightarrow .18 - x = .05$

Forces:

$\tan\theta = \frac{12}{5}$



$\vec{T} = T\frac{5}{13}\vec{i} + T\frac{12}{13}\vec{j}$

NII:

$\rightarrow \quad T\frac{5}{13} + T = mw^2 r$

$\Rightarrow \boxed{\frac{18T}{13} = mw^2(.05)}$ (1)

$\downarrow \quad T\frac{12}{13} - mg = 0.$

$T\frac{12}{13} = mg$

$\Rightarrow \boxed{T = \frac{13mg}{12}}$ (2)

Sub from (2) into (1)

$\Rightarrow \quad \frac{18}{13}\left(\frac{13mg}{12}\right) = mw^2(.05)$

$\Rightarrow \quad \frac{3}{2}g = w^2(.05)$

$\Rightarrow \quad w^2 = 30g \quad \Rightarrow w = \sqrt{30g} = \sqrt{294}$ Rad/s

qed

(b) Energy.

(1a)

Initial

$E_I = \frac{1}{2}mu^2 + 0.$

(2) Typical

$E_B = \frac{1}{2}mv^2 + mgh.$

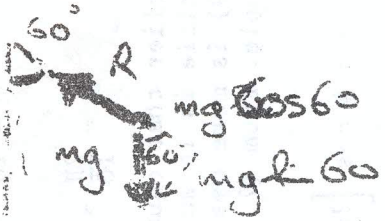
$= \frac{1}{2}mv^2 + mg[a - a\cos 60]$

$= \frac{1}{2}mv^2 + mg\frac{a}{2}$

PCE $\Rightarrow \quad E_I = E_B$

$\Rightarrow \quad \frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\frac{a}{2} \quad \Rightarrow \boxed{mu^2 = mv^2 + mga}$ (1)

forces: Typical position.



Radially, NII $\Rightarrow$

$R - mg\cos 60 = \frac{mv^2}{a}$

$\Rightarrow \boxed{R - \frac{mg}{2} = \frac{mv^2}{a}}$ (2)

Need to eliminate mv^2 from (1) and (2).

$\Rightarrow \quad R - \frac{mg}{2} = \frac{mu^2 - mga}{a}$

$\Rightarrow \quad R - \frac{mg}{2} = \frac{mu^2}{a} - mg \quad \Rightarrow R = \frac{mu^2}{a} - mg + \frac{mg}{2}$

$\Rightarrow R = m\left(\frac{u^2}{a} - \frac{g}{2}\right)$

qed.