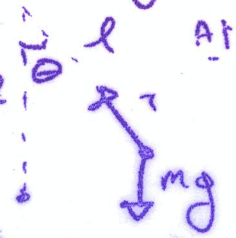
81 Mons

6(a) $T = 2\pi\sqrt{\frac{l}{g}}$.

Forces



At instant Speed = v

in tangential dirn.

NII ⇒ $-mg\sin\theta = m\,\text{accel}$

θ small ⇒ $-mg\theta = m\,\text{accel}$

⇒ $\text{accel} = -g\theta$

⇒ $\text{accel} = -\frac{g}{l}x$

⇒ $\ddot{x} = -\frac{g}{l}x$

SHM with "ω" $= \sqrt{\frac{g}{l}}$

⇒ T periodic time $= \frac{2\pi}{\omega}$

$T = 2\pi\sqrt{\frac{l}{g}}$ qed

---

32 oscillations per minute (=60 secs)

⇒ Periodic time $= \frac{60}{32}$ secs.

[ie 1 oscillation $= \frac{60}{32}$ sec]

⇒ $T_2 = \frac{60}{32}$ secs.

But $T_1 = 2$ secs

% change in length

$= \frac{l_2 - l_1}{l_1} \times \frac{100}{1}\%$

$= \left(\frac{l_2}{l_1} - 1\right) \times \frac{100}{1}\%$.

But $T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow l = \left(\frac{T}{2\pi}\right)^2 g$

⇒ $l_2 = \left(\frac{T_2}{2\pi}\right)^2 g$ and $l_1 = \left(\frac{T_1}{2\pi}\right)^2 g$

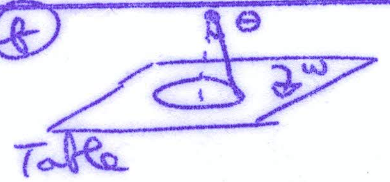
⇒ $\frac{l_2}{l_1} = \frac{T_2^2}{T_1^2}$ (Rest cancel)

$\frac{l_2}{l_1} = \frac{\left(\frac{60}{32}\right)^2}{2^2} = \left(\frac{15}{16}\right)^2$

∴ % change $= \left(\left(\frac{15}{16}\right)^2 - 1\right) \times \frac{100}{1}\%$

$= 12 \cdot 1\%$

(b)

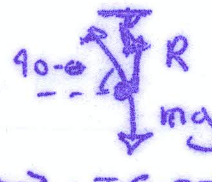


Geometry

$\tan\theta = \frac{r}{\cdot 1}$

⇒ $r = \frac{1}{10}\tan\theta$.

Forces:



$\vec{T} = T\sin\theta\,\vec{i} + T\cos\theta\,\vec{j}$

Equil NII ↕ $R + T\cos\theta - mg = 0$.

Told $R = \frac{mg}{2}$

∴ ↕ ⇒ $\frac{mg}{2} + T\cos\theta - mg = 0$

$$T\cos\theta = \frac{mg}{2} \quad (1)$$

NII ↔ $T\sin\theta = m\omega^2 r$

$$T\sin\theta = m\omega^2\frac{\tan\theta}{10} \quad (2)$$

Want to solve (1) and (2) for ω

$\frac{(2)}{(1)}$ ⇒ $\frac{T\sin\theta}{T\cos\theta} = \frac{\omega^2\left(\frac{\tan\theta}{10}\right)}{\frac{mg}{2}}$

⇒ $\tan\theta = \frac{2}{g}\omega^2\left(\frac{\tan\theta}{10}\right)$

⇒ $1 = \frac{g}{5}\omega^2$

⇒ $\frac{5}{g} = \omega^2$

⇒ $\omega = \sqrt{\frac{5}{g}}$ Rad/sec