

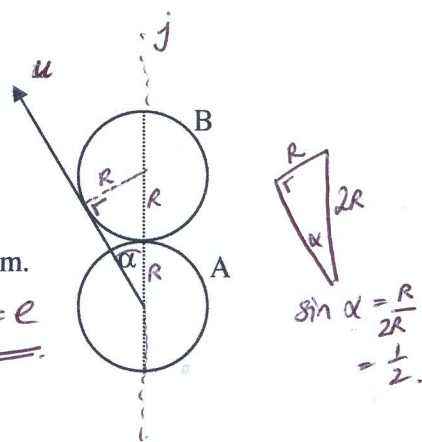
2009 HL

5 (b)

A smooth sphere A, of mass m kg, moving with speed u , collides with a stationary identical smooth sphere B.

The direction of motion of A, before impact, makes an angle α with the line of centres at impact and just touches sphere B, as shown in the diagram.

The coefficient of restitution between the spheres is $\frac{4}{5} = e$.



(i) Show that $\alpha = 30^\circ$.

(ii) Find the direction in which each sphere travels after the collision.

(iii) Find the percentage loss in kinetic energy due to the collision.

Collision only in \vec{j} dir $\Rightarrow \vec{i}$ velocity remains unchanged.

(i)

$$\sin \alpha = \frac{r}{2r} \Rightarrow \alpha = 30^\circ \quad (\text{see diagram})$$

(ii) PCM
in \vec{j} dir.

$$m\left(\frac{u\sqrt{3}}{2}\right) + m(0) = mv_1 + mv_2 \quad (1)$$

NEL
in \vec{j} dir.

$$v_1 - v_2 = -\frac{4}{5}\left(\frac{u\sqrt{3}}{2} - 0\right) \quad (2)$$

$\div m$ (1) $\frac{u\sqrt{3}}{2} = v_1 + v_2$
 (2) $-\frac{2\sqrt{3}u}{5} = v_1 - v_2$
 $u\left(\frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{5}\right) = 2v_1$
 $u\frac{3\sqrt{3}}{20} = v_1$

sub into (1)
 $\frac{u\sqrt{3}}{20} + v_2 = \frac{u\sqrt{3}}{2}$
 $v_2 = \frac{10u\sqrt{3} - u\sqrt{3}}{20}$
 $= \frac{9u\sqrt{3}}{20}$

velocity of A = $-\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}$

direction of A = $\tan^{-1}\left(\frac{\sqrt{3}}{10}\right)$

$\tan^{-1}\left(\frac{u\sqrt{3}}{20}\right)$
 $= \tan^{-1}\left(\frac{\sqrt{3}}{10}\right)$

velocity of B = $0\vec{i} + \frac{9u\sqrt{3}}{20}\vec{j}$

direction of B = along line of centres

$= \tan^{-1}\left(\frac{9u\sqrt{3}}{20}\right)$
 undefined \rightarrow vertically.

5

$u \cos \alpha \vec{j} = \frac{\sqrt{3}}{2}u \vec{j}$
 $-u \sin \alpha \vec{i} = -\frac{u}{2}\vec{i}$

	Before	After
5	$-\frac{u}{2}\vec{i} + \frac{\sqrt{3}}{2}u \vec{j}$	$-\frac{u}{2}\vec{i} + v_1 \vec{j}$
5	$0\vec{i} + 0\vec{j}$	$0\vec{i} + v_2 \vec{j}$

5

5

5

20

(iii)

$\vec{u} = -\frac{u}{2}\vec{i} + \frac{\sqrt{3}}{2}u \vec{j}$
 $|\vec{u}| = \sqrt{\left(-\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}u\right)^2}$
 $= \sqrt{\frac{u^2}{4} + \frac{3u^2}{4}}$
 $= \sqrt{u^2}$

KE before = $\frac{1}{2}mu^2 + \frac{1}{2}M(0)^2$

KE after = $\frac{1}{2}m\left[\frac{u^2}{4} + \frac{3u^2}{400} + \frac{243u^2}{400}\right] = \frac{1}{2}m\frac{346u^2}{400}$

KE lost = $\frac{27}{400}mu^2$

% KE lost = $\frac{\frac{27}{400}mu^2}{\frac{1}{2}mu^2} \times 100 = 13.5\%$

$\left|-\frac{u}{2}\vec{i} + \frac{u\sqrt{3}}{20}\vec{j}\right| = \sqrt{\left(\frac{u}{2}\right)^2 + \frac{u^2}{400}} = \frac{\sqrt{103}u}{20}$
 $\left|0\vec{i} + \frac{9u\sqrt{3}}{20}\vec{j}\right| = \sqrt{0^2 + \frac{243u^2}{400}}$