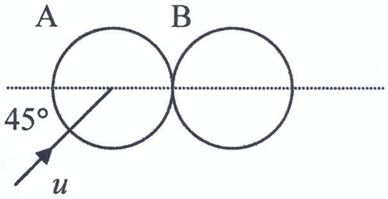
2008 HL

5 (*b*) A smooth sphere A moving with speed u, collides with an identical smooth sphere B which is at rest. ⇒ let mass = m



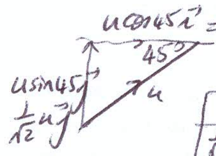
The direction of motion of A, before impact, makes an angle of 45° with the line of centres at the instant of impact.

The coefficient of restitution between the spheres is e.

Show that the direction of motion of A is deflected through an angle α where

$$\tan\alpha = \frac{1+e}{3-e}.$$

Collision is along $\vec{i}$ axis ⇒ $\vec{j}$ velocities unchanged.



Before	Mass	After
$\frac{1}{\sqrt{2}}u\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$	m	$v_1\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$
$0\vec{i} + 0\vec{j}$	m	$v_2\vec{i} + 0\vec{j}$

PCM ($\vec{i}$ dir) $\quad m(u\cos 45) + m(0) = mv_1 + mv_2$ ① 5

NEL ($\vec{i}$ dir) $\quad v_1 - v_2 = -e\,(u\cos 45 - 0)$ ② 5

① ÷ m $\quad \frac{u}{\sqrt{2}} = v_1 + v_2$

$-\frac{eu}{\sqrt{2}} = v_1 - v_2$

$\frac{u(1-e)}{\sqrt{2}} = 2v_1$

$\frac{u(1-e)}{2\sqrt{2}} = v_1$

$v_2 = \frac{u}{\sqrt{2}} - \frac{u(1-e)}{2\sqrt{2}}$

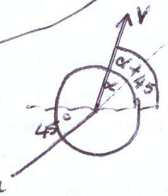
$v_2 = \frac{2u-u+ue}{2\sqrt{2}}$

$v_2 = \frac{u(1+e)}{2\sqrt{2}}$

$$\Rightarrow \quad v_1 = \frac{u}{2\sqrt{2}}(1-e)$$ 5

$v_2 = \frac{u}{2\sqrt{2}}(1+e)$ (not needed)

∴ new velocity of A is $\frac{u}{2\sqrt{2}}(1-e)\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$.



$$\tan(\alpha+45) = \frac{u\sin 45}{v_1} = \frac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}(1-e)}$$

$$= \frac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}(1-e)} = \frac{u}{\sqrt{2}}\cdot\frac{2\sqrt{2}}{u(1-e)}$$

using $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\frac{\tan\alpha + 1}{1-\tan\alpha} = \frac{2}{1-e}$$ $\times(1-e)(1-\tan\alpha)$ 5

$$\tan\alpha + 1 - e\tan\alpha - e = 2 - 2\tan\alpha$$

$$(3-e)\tan\alpha = 1+e$$

$$\tan\alpha = \frac{1+e}{3-e}$$ as required. 5 | 25