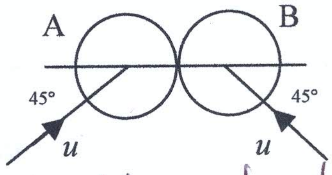
2005 HL

$u\cos 45 = \frac{1}{\sqrt{2}}u\vec{i}$ $\quad$ $u\sin 45 = \frac{1}{\sqrt{2}}u\vec{j}$

$-\frac{1}{\sqrt{2}}u\vec{i}$ $\quad$ $-u\cos 45$ $\quad$ $u\sin 45\vec{j}$ $\quad$ $\frac{1}{\sqrt{2}}u\vec{j}$

5 ***(b)*** A smooth sphere A, of mass m, moving with speed u, collides with an identical (also mass m) smooth sphere B moving with speed u.
The direction of motion of A, before impact, makes an angle 45° with the line of centres at impact.
The direction of motion of B, before impact, makes an angle 45° with the line of centres at impact.
The coefficient of restitution between the spheres is e.



	Before	Mass	After
A	$\frac{1}{\sqrt{2}}u\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$	m	$v_1\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$
B	$-\frac{1}{\sqrt{2}}u\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$	M	$v_2\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$

Collision along $\vec{i}$ axis $\Rightarrow$ $\vec{j}$ velocities unchanged

(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}\left(\frac{4}{3}\right)$.

(i) PCM ($\vec{i}$ dir) $\quad m\dfrac{u}{\sqrt{2}} - m\dfrac{u}{\sqrt{2}} = mv_1 + mv_2$ ①

NEL ($\vec{i}$ dir) $\quad v_1 - v_2 = -e\left(\dfrac{u}{\sqrt{2}} + \dfrac{u}{\sqrt{2}}\right)$ — 5

① $\div m$: $\frac{u}{\sqrt{2}} - \frac{u}{\sqrt{2}} = v_1 + v_2$

$0 = v_1 + v_2$

② $-e\sqrt{2}u = v_1 - v_2$

$-e\sqrt{2}u = 2v_1$

$\frac{-eu}{\sqrt{2}} = v_1$

Sub into ①

$v_2 = -v_1 = \frac{eu}{\sqrt{2}}$.

$\Rightarrow \quad v_1 = -\dfrac{eu}{\sqrt{2}}$ and $v_2 = \dfrac{eu}{\sqrt{2}}$ — 5

Vel of A $= -\frac{eu}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$ $\qquad$ Vel of B $= \frac{eu}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}u\vec{j}$

Speed of A $= \sqrt{\left(-\dfrac{eu}{\sqrt{2}}\right)^2 + \left(\dfrac{u}{\sqrt{2}}\right)^2} = \dfrac{u}{\sqrt{2}}\sqrt{1+e^2}$

$\sqrt{\frac{e^2u^2}{2} + \frac{u^2}{2}} = \frac{\sqrt{u^2(1+e^2)}}{\sqrt{2}} = \frac{u}{\sqrt{2}}\sqrt{(1+e^2)}$.

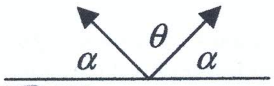
Speed of B $= \dfrac{u}{\sqrt{2}}\sqrt{1+e^2}$ $\quad \left(\sqrt{\left(\frac{eu}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}u\right)^2} = \sqrt{\frac{u^2}{2}(e^2+1)}\right)$ — 5

(ii) $e = \frac{1}{2} \quad \Rightarrow \quad v_1 = -\dfrac{u}{2\sqrt{2}}$ and $v_2 = \dfrac{u}{2\sqrt{2}}$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2(2)}{1-4} = -\dfrac{4}{3}$

$\tan\alpha = \dfrac{\frac{u}{\sqrt{2}}}{\frac{u}{2\sqrt{2}}}$

$\tan\alpha = 2$.



$\theta = 180 - 2\alpha$

$\tan\theta = \dfrac{\tan 180 - \tan 2\alpha}{1 + \tan 180 \tan 2\alpha}$ — 5

$\tan\theta = \dfrac{0 - \left(-\frac{4}{3}\right)}{1+0} = \dfrac{4}{3}$ — 5

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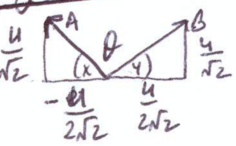
Using slopes



$\tan X = \frac{u}{\sqrt{2}} \times \frac{2\sqrt{2}}{u} = 2$ $\qquad$ $\tan Y = \frac{u}{\sqrt{2}} \times \frac{2\sqrt{2}}{u} = 2$.

slope of A $= -2 = m_1$ $\qquad$ slope of B $= 2 = m_2$

Using angle bet lines formula

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2} = \dfrac{-2-2}{1+(-2)(2)} = \dfrac{-4}{-3} = \dfrac{4}{3}$.