1995(5(a))

(i)

v_1 v_2

(2m) (3m) e

$\overrightarrow{4u}$ $\overleftarrow{(-u)}$

PCM: $2m(4u)+3m(-u) = 2m v_1 + 3m v_2$

$\Rightarrow 8u - 3u = 2v_1 + 3v_2$

$\Rightarrow \boxed{5u = 2v_1 + 3v_2}$ (1) [5]

NLR: $v_2 - v_1 = -e(u_2 - u_1)$

$v_2 - v_1 = -e(-u - 4u)$

$\boxed{v_2 - v_1 = +5ue}$ (2) [5]

Solve (1), (2):

$2v_1 + 3v_2 = 5u$

$-2v_1 + 2v_2 = 10ue$

$5v_2 = 5u + 10ue$

$\Rightarrow \boxed{v_2 = u(1+2e)}$ [5]

$\therefore$ (2) $\Rightarrow u(1+2e) - v_1 = 5ue$

$\Rightarrow u + 2ue - v_1 = 5ue$

$\Rightarrow u - 3ue = v_1$

$\Rightarrow \boxed{(1-3e)u = v_1}$ [5]

(ii) If $e > \frac{1}{3}$ then $v_2 > \left(1+\frac{2}{3}\right)u$

$\Rightarrow v_2 > \frac{5}{3}u$ positive

Also $v_1 < \left(1 - 3\left(\frac{1}{3}\right)\right)u$

$v_1 < 0u$

$v_1 < 0$ negative

$\therefore \vec{v}_1$ and $\vec{v}_2$ have opposite sign and therefore opposite direction after the collision. [5]

(ii) Alternative frac KE loss $= \frac{\Delta KE}{\text{orig KE}}$

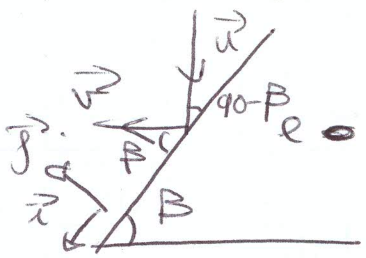
$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{u^2}{u^2} - \frac{v^2}{u^2}$

$= 1 - \left(\frac{v}{u}\right)^2$

$= 1 - (\tan\beta)^2$

$= 1 - e.$

1995(5(b))



$\vec{u} = u\cos(90-\beta)\vec{i} - u\sin(90-\beta)\vec{j}$

Also $\vec{u} = u\sin\beta\vec{i} - u\cos\beta\vec{j}$.

$\vec{v} = v\cos\beta\vec{i} + v\sin\beta\vec{j}$.

Smoothness $\Rightarrow \vec{i}$ cpts unchanged

$u\sin\beta = v\cos\beta$

$\boxed{\tan\beta = \frac{v}{u}}$ (1) [5]

NLR: ($\vec{j}$ dirn)

$v_2 - v_1 = -e(u_2 - u_1)$

$v\sin\beta - 0 = -e(-u\cos\beta - 0)$

$\Rightarrow v\sin\beta = eu\cos\beta$

$\Rightarrow \boxed{\frac{v}{u} = e\cot\beta}$ (2) [5]

(1) $\therefore$ $\tan\beta = e\cot\beta$

$\Rightarrow (\tan\beta)^2 = e$ (3)

$\Rightarrow \boxed{\tan\beta = \sqrt{e}.}$ qed [5]

(ii) fraction of KE loss $= \frac{\Delta KE \text{ in } \vec{j} \text{ dirn}}{\text{Total KE before}}$

$= \frac{\frac{1}{2}m(v\sin\beta)^2 - \frac{1}{2}m(-u\cos\beta)^2}{\frac{1}{2}mu^2}$ [5]

$= \frac{v^2\sin^2\beta - u^2\cos^2\beta}{u^2}$

But $v\sin\beta = eu\cos\beta$

KE loss $= \frac{(eu\cos\beta)^2 - u^2\cos^2\beta}{u^2}$

$= (e^2-1)\cos^2\beta = (e^2-1)\left(\frac{1}{\sec^2\beta}\right)$

$= (e^2-1)\frac{1}{\sec^2\beta}$

$= (e^2-1)\left(\frac{1}{1+\tan^2\beta}\right) \overset{(3)}{=} (e^2-1)\frac{1}{1+(\sqrt{e})^2}$

$= \frac{(e-1)(e+1)}{(1+e)}$

$= e-1.$ [5]