1991

(A)

$v_1$ $\quad$ $4m$ $\qquad\qquad$ $v_2$ $\quad$ $2m$

$u_1 = u$ $\qquad\qquad$ $u_2 = 0$.

Find $v_1, v_2$ in terms of $e$ and $u$.

PCM $\Rightarrow$ $\quad 4mu + 2m(0) = 4mv_1 + 2mv_2$

$$4u = 4v_1 + 2v_2$$

$$\boxed{2u = 2v_1 + v_2} \quad (1)$$

NLE $\Rightarrow$ $\quad v_2 - v_1 = -e(u_2 - u_1)$

$$v_2 - v_1 = -e(0 - u)$$

$$\boxed{v_2 - v_1 = eu} \quad (2)$$

Solve (1), (2):

$$-2u = -2v_1 - v_2$$

$$eu = -v_1 + v_2$$

$$eu - 2u = -3v_1$$

$$(e-2)u = -3v_1$$

$$\boxed{\left(\frac{2-e}{3}\right)u = v_1}$$

$\therefore$ (1) $\Rightarrow 2u = 2\left(\frac{2-e}{3}\right)u + v_2$

$$\Rightarrow 2u = \frac{4}{3}u - \frac{2eu}{3} + v_2$$

$$\Rightarrow \frac{6u}{3} - \frac{4u}{3} + \frac{2eu}{3} = v_2$$

$$\Rightarrow \frac{2u}{3} + \frac{2eu}{3} = v_2$$

$$\Rightarrow v_2 = \frac{u(2+2e)}{3} \Rightarrow \boxed{v_2 = \frac{2u(1+e)}{3}}$$

ΔKE: KE before: $\frac{1}{2}(4m)u^2 + \frac{1}{2}2m0^2 = 2mu^2$.

KE after: $\frac{1}{2}(4m)v_1^2 + \frac{1}{2}2mv_2^2$

$$= 2m\left[\left(\frac{2-e}{3}\right)u\right]^2 + m\left(\frac{2u(1+e)}{3}\right)^2$$

$$= mu^2\frac{2}{9}[2-e]^2 + \frac{4mu^2(1+e)^2}{9}$$

$$= \frac{mu^2}{9}\left[2(4-4e+e^2) + 4[1+2e+e^2]\right]$$

$$= \frac{mu^2}{9}[8-8e+2e^2+4+8e+4e^2]$$

$$= \frac{mu^2}{9}[12+6e^2]$$

$$= \frac{2}{3}mu^2[2+e^2]$$

ΔKE: $\quad 2mu^2 - \frac{2mu^2(2+e^2)}{3} = mu^2\left[2 - \frac{4}{3} - \frac{2}{3}e^2\right] = \frac{2mu^2(1-e^2)}{3}$.