

1988

Collision 1 A/B

$$v_1 = ? \quad v_2 = ?$$

$$\textcircled{4} \textcircled{8} \quad e = \frac{1}{4}$$

$$u_1 = u \quad u_2 = 0$$

PCM \Rightarrow

$$4u + 0 = 4v_1 + 8v_2$$

$$\boxed{u = v_1 + 2v_2} \quad \textcircled{1}$$

NLR \Rightarrow

$$v_2 - v_1 = -\frac{1}{4}(0 - u)$$

$$\Rightarrow \boxed{4v_2 - 4v_1 = u} \quad \textcircled{2}$$

Solve $\textcircled{1}$ and $\textcircled{2}$ for v_1 and v_2 in terms of u

$$\textcircled{1} \times 4 \Rightarrow 4v_1 + 8v_2 = 4u$$

$$-4v_1 + 4v_2 = u$$

$$\Rightarrow 12v_2 = 5u$$

$$\Rightarrow \boxed{v_2 = \frac{5u}{12}}$$

$$\therefore u = v_1 + 2\left(\frac{5u}{12}\right)$$

$$\Rightarrow \frac{12u}{12} = v_1 + \frac{10u}{12}$$

$$\frac{2u}{12} = v_1$$

$$\boxed{\frac{u}{6} = v_1}$$

\therefore A travels towards wall with speed $\frac{u}{6}$

B travels towards wall with speed $\frac{5u}{12}$

Collision 2 B/Wall

$$w \quad | \quad 0$$

$$\textcircled{8} \quad | \quad e = ?$$

$$\frac{5u}{12} \quad | \quad 0$$

NLR \Rightarrow

$$w - 0 = -e\left(\frac{5u}{12} - 0\right)$$

$$\Rightarrow w = -\frac{5eu}{12}$$

To follow this solution (i) proceed as follows: Collision 1 then Collision 2 then Collision 3 then answer to (ii) then to (i) and then to (iii).

Collision 3 A/B again

$$v_1 = d \quad v_2 = 0$$

$$\textcircled{4} \textcircled{8} \quad e = \frac{1}{4}$$

$$u_1 = \frac{u}{6} \quad u_2 = -\frac{5u}{12}$$

PCM \Rightarrow

$$4\left(\frac{1}{6}u\right) + 8\left(-\frac{5u}{12}\right) = 4d + 0$$

$$\Rightarrow \frac{1}{6}u - \frac{10}{12}u = d$$

$$\Rightarrow \boxed{u - 5ue = 6d} \quad \textcircled{3}$$

NLR \Rightarrow

$$0 - d = -\frac{1}{4}\left(-\frac{5ue}{12} - \frac{1}{6}u\right)$$

$$\Rightarrow 4d = -\frac{5ue}{12} - \frac{1}{6}u$$

$$\Rightarrow \boxed{48d = -5ue - 2u} \quad \textcircled{4}$$

we answered (ii) first!

$$\text{From } \textcircled{3} \Rightarrow u - 5ue = 6d$$

$$\textcircled{4} \Rightarrow 2u + 5ue = -48d$$

$$\text{Add} \quad 3u = -42d$$

$$\Rightarrow \boxed{\frac{-u}{14} = d}$$

velocity of A after all collisions is $-\frac{u}{14} \hat{i}$.

Answer (i)

$$\textcircled{3} \Rightarrow u - 5ue = 6\left(\frac{-u}{14}\right)$$

$$\Rightarrow 1 - 5e = -\frac{3}{7}$$

$$\Rightarrow -5e = -\frac{10}{7}$$

$$\Rightarrow \boxed{e = \frac{2}{7}}$$

Answer (iii)

$$\text{KE before all collisions} = \frac{1}{2}4u^2 + \frac{1}{2}8(0)^2 = 2u^2$$

$$\text{KE after all collisions} = \frac{1}{2}4d^2 + \frac{1}{2}8(0)^2$$

$$= 2d^2$$

$$= 2\left(\frac{u}{14}\right)^2$$

$$= \frac{u^2}{49}$$

$$\Rightarrow \Delta \text{KE} = 2u^2 - \frac{u^2}{49} = u^2 \left[\frac{196}{49} - \frac{1}{49} \right] = \frac{u^2 195}{49}$$