

1986 (ctd)

Q5B

$$v_1 = c\vec{i} + b\vec{j}$$

$$\vec{v}_2 = 0\vec{i} + 0\vec{j}$$

(4)

(2)

$$e = \frac{1}{2}$$

$$\vec{u}_1 = a\vec{i} + b\vec{j}$$

$$\vec{u}_2 = x\vec{i} + 0\vec{j}$$

Smoothness \Rightarrow \vec{v} cpts unchanged before and after

PCM:

$$(\vec{i} \text{ dir})$$

\Rightarrow

$$4a + 2x = 4c + 2(0)$$

$$\underline{2a + x = 2c}$$

(1)

NLR

$$(\vec{i} \text{ dir})$$

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$0 - c = -\frac{1}{2}(x - a)$$

$$\Rightarrow -2c = -x + a$$

$$\Rightarrow \underline{-a + x = 2c}$$

(2)

(i) Equations (1) and (2) are inconsistent unless $\boxed{a=0}$

$$\therefore a=0 \Rightarrow \vec{u}_1 = 0\vec{i} + b\vec{j} \text{ and } \vec{u}_2 = x\vec{i} + 0\vec{j}$$

$$\Rightarrow \vec{u}_1 \cdot \vec{u}_2 = 0(x) + b(0) = 0$$

$$\Rightarrow \vec{u}_1 \perp \vec{u}_2$$

(ii) If $\boxed{a=0}$ (as proven!) (1) $\Rightarrow x=2c$

$$\begin{aligned} \text{KE Loss for 2 kg mass} &= \frac{1}{2} (0)^2 - \frac{1}{2} 2(x)^2 \\ (\vec{i} \text{ cpts only}) &= 0 - x^2 \end{aligned}$$

$$= 0 - (2c)^2$$

$$= -4c^2$$

$$\begin{aligned} \text{REGAIN for 4 kg mass} &= \frac{1}{2} 4c^2 - \frac{1}{2} 4(0)^2 \\ (\vec{i} \text{ cpts only}) &= 2c^2 - 0 \end{aligned}$$

$$= 2c^2$$

$$= \frac{1}{2} \text{ loss of 2 kg mass.}$$