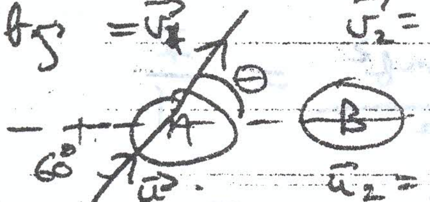
(H 1985) LAWS TEXT.

$a\vec{i}+b\vec{j} = \vec{v_1}$ $\qquad \vec{v_2} = X\vec{i}+0\vec{j}$.



$\vec{u_2} = 0\vec{i}+0\vec{j}$

$\vec{u} = \frac{u}{2}\vec{i} + \frac{\sqrt{3}}{2}u\vec{j}$

N.B. $\tan\theta = \frac{b}{a}$.

Smoothness of spheres ⇒

$\boxed{b = \frac{\sqrt{3}}{2}u}$ (1)

and $\vec{j}$ cpt of $v_2 = 0$.

NLR along line of centres:

$a - X = -e\left(\frac{u}{2} - 0\right)$

$\boxed{a - X = -e\frac{u}{2}}$ (2)

PCM (along centres) ⇒ $m\frac{u}{2} + m0 = ma + mX$

$\Rightarrow \boxed{\frac{u}{2} = a + X}$ (3)

(i) $\tan\theta = ----$

Need a and b. Solve (2) and (3) to get a.

$2a = \frac{u}{2} - e\frac{u}{2} \Rightarrow \boxed{a = \frac{1}{2}\frac{u}{2}(1-e)}$ (4)

$\frac{(1)}{(4)} \Rightarrow \frac{\frac{\sqrt{3}}{2}u}{\frac{u}{4}[1-e]} = \frac{b}{a} \Rightarrow \frac{2\sqrt{3}}{[1-e]} = \tan\theta$. q.e.d

(ii) KE fractional loss (MAX LOSS.) $= \frac{\text{KE old} - \text{KE New}}{\text{KE old}}$

explained at end.

$= \frac{\frac{1}{2}mu^2 - \left[\frac{1}{2}ma^2 + \frac{1}{2}mb^2 + \frac{1}{2}mX^2\right]}{\frac{1}{2}mu^2}$

get a, b, X in term of u by (1), (2), (3)

solve (2) and (3) to get by subtracting

$\therefore 2X = \frac{u}{2}[1+e] \Longrightarrow \boxed{X = \frac{u}{4}[1+e]}$ (5)

$\boxed{\text{MAX KE LOSS} \Rightarrow e = 0}$ $\therefore b = \frac{\sqrt{3}}{2}u,\ a \underset{(4)}{=} \frac{u}{4},\ X \underset{(5)}{=} \frac{u}{4}$.

$\therefore$ KE frac loss $= \frac{u^2 - \left[\left(\frac{u}{4}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2 + \left(\frac{u}{4}\right)^2\right]}{u^2}$

Cancelling u^2

$= \frac{1 - \left[\frac{1}{16} + \frac{3}{4} + \frac{1}{16}\right]}{1} = 1 - \frac{14}{16} = \frac{2}{16} = \frac{1}{8}$

Max KE % Loss $= 12\frac{1}{2}\%$.

qed