HLC 1993 Q3

(a) 

If $R = 3H$ prove $\tan\beta = \frac{4}{3}$. 5

$\vec{u} = u\cos\beta\,\vec{i} + u\sin\beta\,\vec{j}$, $\vec{g} = 0\vec{i} - g\vec{j}$ $\Rightarrow$ $\vec{v}(t) = u\cos\beta\,\vec{i} + (u\sin\beta - gt)\vec{j}$ 5

$\vec{r}(t) = u\cos\beta\, t\,\vec{i} + \left(u\sin\beta\, t - \frac{gt^2}{2}\right)\vec{j}$. 5

First find T: At q $(\vec{r})_{\vec{j}} = 0 \Rightarrow u\sin\beta\, t - g\frac{t^2}{2} = 0$

$\Rightarrow t\left(u\sin\beta - \frac{g}{2}t\right) = 0$

$\Rightarrow t = 0$ or $u\sin\beta - \frac{g}{2}t = 0$

$\Rightarrow t = 0$ or $T = \frac{2u\sin\beta}{g}$ 5

$\therefore$ Time to reach max height $= \frac{1}{2}T = \frac{u\sin\beta}{g}$.

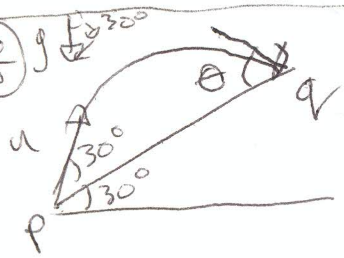
$\therefore H = \left(\vec{r}\left(\frac{u\sin\beta}{g}\right)\right)_{\vec{j}} = u\sin\beta\left(\frac{u\sin\beta}{g}\right) - \frac{g}{2}\left(\frac{u\sin\beta}{g}\right)^2 = \frac{u^2\sin^2\beta}{g} - \frac{u^2\sin^2\beta}{2g} = \frac{u^2\sin^2\beta}{2g}$ 3

$\therefore R = \left(\vec{r}\left(\frac{2u\sin\beta}{g}\right)\right)_{\vec{i}} = u\cos\beta\left(\frac{2u\sin\beta}{g}\right) = \frac{u^2\, 2\sin\beta\cos\beta}{g}$. 2

Told $R = 3H \Rightarrow \frac{u^2\, 2\sin\beta\cos\beta}{g} = 3\left(\frac{u^2\sin^2\beta}{2g}\right)$

$\Rightarrow 4\sin\beta\cos\beta = 3(\sin^2\beta)$

$\Rightarrow \frac{4}{3} = \frac{\sin^2\beta}{\cos\beta\sin\beta} \Rightarrow \boxed{\tan\beta = \frac{4}{3}}$ 5

(b) 

(i) $\vec{u} = u\cos 30\,\vec{i} + u\sin 30\,\vec{j} = \frac{\sqrt{3}}{2}u\,\vec{i} + \frac{u}{2}\vec{j}$ m/s

$\vec{g} = -g\sin 30\,\vec{i} - g\cos 30\,\vec{j} = -\frac{g}{2}\vec{i} - g\frac{\sqrt{3}}{2}\vec{j}$ m/s²

$\therefore \vec{v}(t) = \left(\frac{\sqrt{3}}{2}u - \frac{g}{2}t\right)\vec{i} + \left(\frac{u}{2} - \frac{g\sqrt{3}}{2}t\right)\vec{j}$ m/s 5

$\therefore \vec{r}(t) = \left(\frac{\sqrt{3}}{2}ut - \frac{g}{4}t^2\right)\vec{i} + \left(\frac{u}{2}t - \frac{g\sqrt{3}}{4}t^2\right)\vec{j}$ m 5

(ii) $\tan\theta = \frac{|\vec{v}(T)_{\vec{j}}|}{\vec{v}(T)_{\vec{i}}}$. First find T, time in flight 5

At q: $(\vec{r}(T))_{\vec{j}} = 0 \Rightarrow \frac{u}{2}t - \frac{g\sqrt{3}}{4}t^2 = 0$

$\Rightarrow t = 0$ or $\left(\frac{u}{2} - \frac{g\sqrt{3}}{4}t\right) = 0$

$\Rightarrow t = 0$ or $T = \frac{u\,4}{2g\sqrt{3}} = \frac{2u}{g\sqrt{3}}$ 5

$\therefore \tan\theta = \frac{\left|\frac{u}{2} - \frac{g\sqrt{3}}{2}\left(\frac{2u}{g\sqrt{3}}\right)\right|}{\frac{\sqrt{3}u}{2} - \frac{g}{2}\left(\frac{2u}{g\sqrt{3}}\right)} = \frac{\left|\frac{u}{2} - u\right|}{\frac{\sqrt{3}}{2}u - \frac{1}{\sqrt{3}}u} = \frac{\left|\frac{u}{2}\right|}{u\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}}$.

$\Rightarrow \tan\theta = \frac{0.5}{0.8660254 - 0.5773503} = \frac{.5}{.2886751} = 1.732$ 5

$\Rightarrow \boxed{\theta = 60^\circ}$