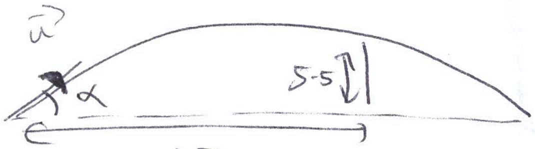
1992 HLC – Q3



$\vec{u} = 6\sqrt{g}$

$\vec{u} = 6\sqrt{g}\cos\alpha\,\vec{i} + 6\sqrt{g}\sin\alpha\,\vec{j}$

$\vec{g} = 0\,\vec{i} - g\,\vec{j}$

$\vec{r}(t) = (6\sqrt{g}\cos\alpha\, t)\,\vec{i} + \left(6\sqrt{g}\sin\alpha\, t - \frac{g}{2}t^2\right)\vec{j}$.

To just clear obstacle (find the least α)

$(\vec{r})_{\vec{i}} = 18$

$6\sqrt{g}\cos\alpha\, t = 18$

$t = \dfrac{3}{\sqrt{g}\cos\alpha}$

and $(\vec{r})_{\vec{j}} = 5\cdot5$

$\Rightarrow 6\sqrt{g}\sin\alpha\left(\dfrac{3}{\sqrt{g}\cos\alpha}\right) - \dfrac{g}{2}\,\dfrac{9}{g\cos^2\alpha} = 5\cdot5$

$= 5\cdot5$

Must solve for α. TRIG EQN!

$\Rightarrow 18\tan\alpha - \dfrac{4\cdot5}{\cos^2\alpha}$

$\Rightarrow 18\tan\alpha - 4\cdot5(\sec^2\alpha) = 5\cdot5$

$\Rightarrow 18\tan\alpha - 4\cdot5(1+\tan^2\alpha) = 5\cdot5$

$\Rightarrow 36\tan\alpha - 9 - 9\tan^2\alpha = 11$

$\Rightarrow 9\tan^2\alpha - 36\tan\alpha + 20 = 0$

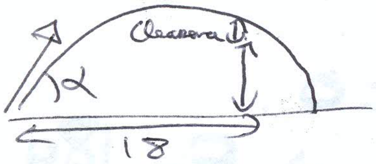
Let $x = \tan\alpha$ $\Rightarrow 9x^2 - 36x + 20 = 0$

$\Rightarrow (3x-2)(3x-10) = 0$

$\Rightarrow x = \frac{2}{3}$ or $x = \frac{10}{3}$

$\Rightarrow \tan\alpha = \frac{2}{3}$ or $\tan\alpha = \frac{10}{3}$.

(ii) clearance height $= r_{\vec{j}} - 5\cdot5$.



To maximise clearance with respect to the angle of projection α:

$\dfrac{d\,\text{Clearance}}{d\alpha} = 0$ at max.

$r_{\vec{j}} = 6\sqrt{g}\sin\alpha\,\dfrac{3}{\sqrt{g}\cos\alpha} - \dfrac{g\,9}{2g\cos^2\alpha} = 18\tan\alpha - 4\cdot5(1+\tan^2\alpha)$

$\Rightarrow \text{Clearance} = 18\tan\alpha - 4\cdot5(1+\tan^2\alpha) - 5\cdot5$.

$\dfrac{d}{d\alpha}(\text{Clearance}) = 18\sec^2\alpha - 4\cdot5(0 + 2\tan\alpha\sec^2\alpha) - 0$

$\dfrac{d\,\text{Clearance}}{d\alpha} = 18\sec^2\alpha - 9\tan\alpha\sec^2\alpha$.

$\dfrac{d\,\text{Clearance}}{d\alpha} = 0 \Rightarrow 18\sec^2\alpha - 9\tan\alpha\sec^2\alpha = 0$

$\Rightarrow 9\sec^2\alpha(2 - \tan\alpha) = 0$

$\Rightarrow \sec\alpha = 0$ or $2 - \tan\alpha = 0$

$\Rightarrow \boxed{\tan\alpha = 2}$ qed.