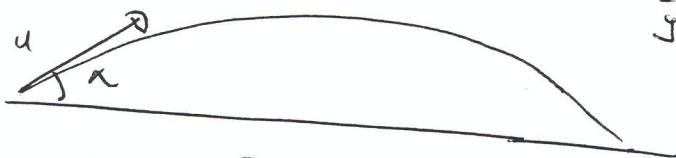


1989



$$\vec{g} = 0\vec{i} - g\vec{j}$$

(i)

$$\vec{v}(t) = u \cos \alpha \vec{i} + (u \sin \alpha - gt) \vec{j}$$

$$\vec{r}(t) = u \cos \alpha t \vec{i} + \left(u \sin \alpha t - \frac{gt^2}{2} \right) \vec{j}$$

Time in flight

$$(\vec{r}(t))_{\vec{j}} = 0$$

$$\Rightarrow u \sin \alpha T - \frac{g}{2} T^2 = 0$$

$$\Rightarrow T = 0 \quad \text{or} \quad u \sin \alpha - \frac{g}{2} T = 0$$

$$\Rightarrow T = 0 \quad \text{or} \quad T = \frac{2u \sin \alpha}{g}$$

Range:

$$R = (\vec{r}(t))_{\vec{i}}$$

$$= u \cos \alpha \left(\frac{2u \sin \alpha}{g} \right)$$

$$= \frac{u^2 2 \sin \alpha \cos \alpha}{g}$$

$$= \frac{u^2 \sin 2\alpha}{g}$$

Range max:

$$\frac{dR}{d\alpha} = 0 \Rightarrow \frac{u^2}{g} \frac{d \sin 2\alpha}{d\alpha} = 0$$

$$\Rightarrow 2 \cos 2\alpha = 0$$

$$\Rightarrow \cos 2\alpha = 0$$

$$\Rightarrow 2\alpha = 90$$

$$\Rightarrow \alpha = 45^\circ$$

(ii)

$$R = \frac{u^2}{2g} \Rightarrow \frac{u^2}{2g} = \frac{u^2 \sin 2\alpha}{g}$$

$$\Rightarrow \frac{1}{2} = \sin 2\alpha$$

$$\Rightarrow 2\alpha = \frac{\pi}{6}, \text{ or } 150$$

$$\Rightarrow 2\alpha = 30 \text{ or } 150$$

$$\Rightarrow \alpha = 15 \text{ or } 75^\circ$$

(iii)

$$\text{Time to reach greatest height: } \frac{T}{2} = \frac{u \sin \alpha}{g}$$

$$\Rightarrow \text{greatest height } H = (\vec{r}(\frac{T}{2}))_{\vec{j}} = u \sin \alpha \left(\frac{u \sin \alpha}{g} \right) - \frac{g}{2} \left(\frac{u \sin \alpha}{g} \right)^2$$

$$= \frac{u^2 \sin^2 \alpha}{g} - \frac{u^2 \sin^2 \alpha}{2g}$$

$$= \frac{u^2 \sin^2 \alpha}{2g}$$

$$H : R = 2 : 5 \Rightarrow$$

$$\frac{\frac{u^2 \sin^2 \alpha}{2g}}{\frac{u^2 \sin 2\alpha}{g}} = \frac{2}{5}$$

$$\Rightarrow \frac{\sin^2 \alpha}{2 \sin 2\alpha} = \frac{2}{5} \Rightarrow \tan \alpha = \frac{2}{5} \Rightarrow \alpha = \tan^{-1} \frac{2}{5}$$