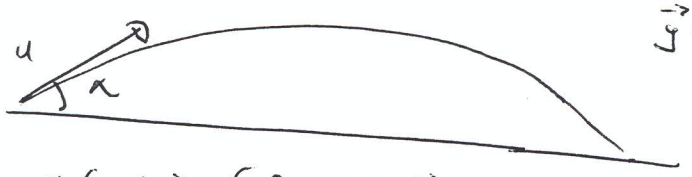
1989



$\vec{g} = 0\vec{i} - g\vec{j}$

(i)

$\vec{v}(t) = u\cos\alpha\,\vec{i} + (u\sin\alpha - gt)\vec{j}$

$\vec{r}(t) = u\cos\alpha\,t\,\vec{i} + \left(u\sin\alpha\,t - \frac{gt^2}{2}\right)\vec{j}$

Time in flight $\left(\vec{r}(t)\right)_{\vec{j}} = 0$

$\Rightarrow u\sin\alpha\,T - \frac{g}{2}T^2 = 0$

$\Rightarrow T = 0$ or $u\sin\alpha - \frac{g}{2}T = 0$

$\Rightarrow T = 0$ or $T = \frac{2u\sin\alpha}{g}$.

Range:

$R = \left(r(t)\right)_{\vec{i}}$

$= u\cos\alpha\left(\frac{2u\sin\alpha}{g}\right)$

$= \frac{u^2 2\sin\alpha\cos\alpha}{g}$

$= \frac{u^2\sin 2\alpha}{g}$

Range max: $\frac{dR}{d\alpha} = 0 \Rightarrow \frac{u^2}{g}\frac{d}{d\alpha}\sin 2\alpha = 0$

$\Rightarrow 2\cos 2\alpha = 0$

$\Rightarrow \cos 2\alpha = 0$

$\Rightarrow 2\alpha = 90$

$\Rightarrow \alpha = 45°$

(ii) $R = \frac{u^2}{2g} \Rightarrow \frac{u^2}{2g} = \frac{u^2\sin 2\alpha}{g}$

$\Rightarrow \frac{1}{2} = \sin 2\alpha$.

$\Rightarrow 2\alpha = \frac{\pi}{6}$, or 150.

$\Rightarrow 2\alpha = 30$ or 150

$\Rightarrow \alpha = 15$ or $75°$.

(iii) Time to reach greatest height: $\frac{T}{2} = \frac{u\sin\alpha}{g}$.

$\Rightarrow$ Greatest Height $H = \left(r\left(\frac{T}{2}\right)\right)_{\vec{j}} = u\sin\alpha\left(\frac{u\sin\alpha}{g}\right) - \frac{g}{2}\left(\frac{u\sin\alpha}{g}\right)^2$

$= \frac{u^2\sin^2\alpha}{g} - \frac{u^2\sin^2\alpha}{2g}$

$= \frac{u^2\sin^2\alpha}{2g}$.

$H : R = 2 : 5 \Rightarrow \frac{\frac{u^2\sin^2\alpha}{2g}}{\frac{u^2\sin 2\alpha}{g}} = \frac{2}{5}$

$\Rightarrow \frac{\sin^2\alpha}{2\sin 2\alpha} = \frac{2}{5} \Rightarrow \tan\alpha = \frac{8}{5} \Rightarrow \alpha = \tan^{-1}\frac{8}{5}$.