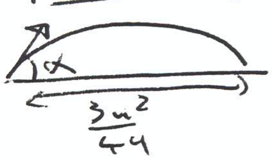
1988 Q3(a)



$\vec u = u\cos\alpha\,\vec i + u\sin\alpha\,\vec j$
$\vec g = 0\vec i - 9.8\vec j$
$\Rightarrow \vec r(t) = u\cos\alpha\, t\,\vec i + \left(u\sin\alpha\, t - \frac{gt^2}{2}\right)\vec j$ (5)

$(\vec r(t))_{\vec j} = 0 \Rightarrow u\sin\alpha\, t - \frac{g}{2}t^2 = 0$

$\Rightarrow T = \frac{2u\sin\alpha}{g}$ (5)

$(\vec r(T))_{\vec i} = \frac{3u^2}{49} \Rightarrow u\cos\alpha\,\frac{2u\sin\alpha}{g} = \frac{3u^2}{49}$

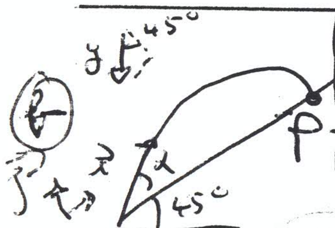
$\Rightarrow \frac{u^2\sin 2\alpha}{9.8} = \frac{3u^2}{49}$ (5)

$\Rightarrow \sin 2\alpha = \frac{3}{5}$.

$\Rightarrow \sin 2\alpha = .6$, $2\alpha = 36°52'$ $\Rightarrow \alpha = 18°26'$ or $90 - 18°26'$

$\Rightarrow \alpha = 18°26'$ or $71°34'$. (5)

(b)



Need Velocity at P

$\tan\alpha = \frac{5}{12} \Rightarrow \cos\alpha = \frac{12}{13}$, $\sin\alpha = \frac{5}{13}$

PROJECTILE

$\Rightarrow \vec u = 12u\vec i + 5u\vec j$

$\Rightarrow \vec g = -g\frac{1}{\sqrt2}\vec i - \frac{g}{\sqrt2}\vec j$

$\Rightarrow \vec r(t) = \left(12ut - \frac{g}{2\sqrt2}t^2\right)\vec i + \left(5ut - \frac{g}{2\sqrt2}t^2\right)\vec j$ (5)

$\Rightarrow \vec v(t) = \left(12u - \frac{g}{\sqrt2}t\right)\vec i + \left(5u - \frac{g}{\sqrt2}t\right)\vec j$

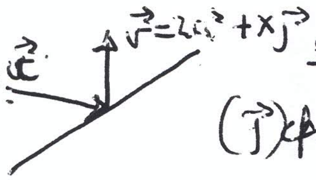
At P $(\vec r(t))_{\vec j} = 0 \Rightarrow 5ut - \frac{g}{2\sqrt2}t^2 = 0 \Rightarrow t = 0$ or $T = \frac{u10\sqrt2}{g}$. (5)

Need the velocity at p: $\vec v(T) = \left(12u - \frac{g}{\sqrt2}\left[\frac{u10\sqrt2}{g}\right]\right)\vec i + \left(5u - \frac{g}{\sqrt2}\left(\frac{u10\sqrt2}{g}\right)\right)\vec j$

$\vec v(T) = (12u - 10u)\vec i + (5u - 10u)\vec j$

$\vec v(T) = 2u\vec i - 5u\vec j$ (5)

Collision: $2u\vec i - 5u\vec j$, $e = \frac{2}{5}$



Smoothness ⇒ $\vec i$ cpt unchanged.

($\vec j$) cpt NLR ⇒ $v_2 - v_1 = -e(u_2 - u_1)$

$\Rightarrow x = -\frac{2}{5}(-5u - 0)$

$\Rightarrow x = 2u$. (5)

⇒ Velocity on rebound is $2u\vec i + 2u\vec j$.

This is a vector making angle $\tan^{-1}\frac{2u}{2u} = 45°$ with plane and so $90°$ with horizontal. (5)

⇒ Particle rises vertically from p

⇒ Particle will fall to p again on second bounce