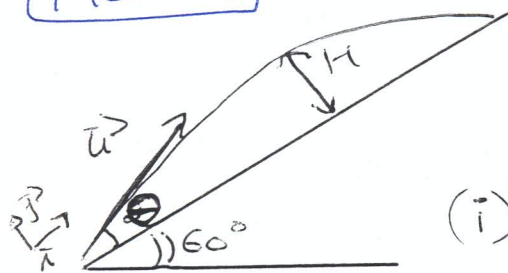


1985 Haws



$$g = g \cos 60^\circ \hat{i} - g \sin 60^\circ \hat{j}$$

$$\vec{u} = u \cos \theta \hat{i} + u \sin \theta \hat{j}$$

$$\vec{v} = -g \sin 60^\circ \hat{i} - g \cos 60^\circ \hat{j}$$

$$\vec{g} = -\frac{\sqrt{3}}{2} g \hat{i} - \frac{g}{2} \hat{j}$$

$$(i) \vec{v}(t) = (u \cos \theta - g \frac{\sqrt{3}}{2} t) \hat{i} + (u \sin \theta - \frac{g}{2} t) \hat{j}$$

$$\vec{r}(t) = (u \cos \theta t - \frac{g \sqrt{3}}{4} t^2) \hat{i} + (u \sin \theta t - \frac{g}{4} t^2) \hat{j}$$

(ii) Finding H: Find the time to reach the max height by either finding the time in flight and halving it or noting at max. height

$$(\vec{v})_j = 0 \Rightarrow u \sin \theta - \frac{g}{2} t = 0 \Rightarrow t^* = \frac{2u \sin \theta}{g}$$

$$\Rightarrow H = (\vec{r}(t^*))_j = u \sin \theta \left( \frac{2u \sin \theta}{g} \right) - \frac{g}{4} \left( \frac{2u \sin \theta}{g} \right)^2$$

$$= \frac{2u^2 \sin^2 \theta}{g} - \frac{u^2 \sin^2 \theta}{g}$$

$$H = \frac{u^2 \sin^2 \theta}{g}$$

$$(iii) \text{ If } (\vec{r})_j = H \sin^2 \theta \Rightarrow (\vec{r})_j = \frac{u^2 \sin^4 \theta}{g}$$

$$\text{At the times } t \Rightarrow u \sin \theta t - \frac{g}{4} t^2 = \frac{u^2 \sin^4 \theta}{g}$$

This is a quadratic equation in t  $\Rightarrow \frac{g}{4} t^2 - u \sin \theta t + \frac{u^2 \sin^4 \theta}{g} = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow t = \frac{u \sin \theta \pm \sqrt{(u \sin \theta)^2 - 4 \left( \frac{g}{4} \right) \frac{u^2 \sin^4 \theta}{g}}}{2 \left( \frac{g}{4} \right)}$$

$$\Rightarrow t = \frac{u \sin \theta \pm \sqrt{u^2 \sin^2 \theta - u^2 \sin^4 \theta}}{\frac{g}{2}}$$

$$\Rightarrow t = \frac{u \sin \theta \pm u \sin \theta \sqrt{1 - \sin^2 \theta}}{\frac{g}{2}}$$

$$\Rightarrow t = \frac{2}{g} [u \sin \theta \pm u \sin \theta \sqrt{\cos^2 \theta}]$$

times at which particle is at height  $H \sin^2 \theta$

$$\Rightarrow \text{Time above} = \frac{2}{g} [u \sin \theta + u \sin \theta \cos \theta]$$

$$= \frac{2}{g} [u \sin \theta + u \sin \theta \cos \theta] - \frac{2}{g} [u \sin \theta - u \sin \theta \cos \theta]$$

$$= \frac{2u \sin \theta \cos \theta}{g} + \frac{2u \sin \theta \cos \theta}{g}$$

$$= \frac{4u \sin \theta \cos \theta}{g} = \frac{2u \sin 2\theta}{g}$$

$$\text{Time above } H \sin^2 \theta = \frac{2u \sin 2\theta}{g}$$