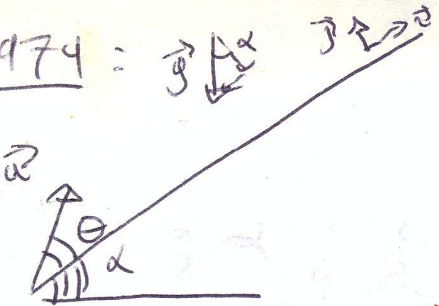
1974 :



$\vec{u} = u\cos\theta\,\vec{i} + u\sin\theta\,\vec{j}$

$\vec{g} = -g\sin\alpha\,\vec{i} - g\cos\alpha\,\vec{j}$.

$\vec{v}(t) = (u\cos\theta - g\sin\alpha\, t)\,\vec{i} + (u\sin\theta - g\cos\alpha\, t)\,\vec{j}$.

$\vec{r}(t) = \left(u\cos\theta\, t - g\frac{\sin\alpha}{2}t^2\right)\vec{i} + \left(u\sin\theta\, t - g\frac{\cos\alpha\, t^2}{2}\right)\vec{j}$.

(i) find t. At q $\quad (\vec{r}(t))_{\vec{j}} = 0$

$\Rightarrow u\sin\theta\, t - g\frac{\cos\alpha}{2}t^2 = 0$

$\Rightarrow t = 0$ or $u\sin\theta - g\frac{\cos\alpha}{2}t = 0$

$u\sin\theta = g\frac{\cos\alpha}{2}t$

$\frac{2u\sin\theta}{g\cos\alpha} = T$

(ii) Range $= \vec{r}(T)\vec{i} = \left(u\cos\theta\, T - g\frac{\sin\alpha}{2}T^2\right)$

Range(θ) $= u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - g\frac{\sin\alpha}{2}\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$

[α cannot vary can't move the mountain!]

$= \frac{2u^2\sin\theta\cos\theta}{g\cos\alpha} - g\frac{\sin\alpha}{2}\,\frac{2u\sin\theta}{g\cos\alpha}\,\frac{2u\sin\theta}{g\cos\alpha}$

$= \frac{2u^2\sin\theta\cos\theta}{g\cos\alpha} - \frac{2u^2\sin^2\theta\sin\alpha}{g\cos^2\alpha}$

$= \frac{2u^2\sin\theta}{g\cos^2\alpha}\left[\cos\theta\cos\alpha - \sin\theta\sin\alpha\right]$

Range$(\theta) = \frac{2u^2\sin\theta}{g\cos^2\alpha}\left[\cos(\theta+\alpha)\right]$

Range max $\Rightarrow \frac{dR(\theta)}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}\right] = 0$

$\frac{2u^2}{g\cos^2\alpha}$ constant $\Rightarrow \frac{2u^2}{g\cos^2\alpha}\,\frac{d(\sin\theta\cos(\theta+\alpha))}{d\theta} = 0$

$\Rightarrow \frac{d(\sin\theta\cos(\theta+\alpha))}{d\theta} = 0$

Product Rule $\Rightarrow \sin\theta[-\sin(\theta+\alpha)] + \cos(\theta+\alpha)[\cos\theta] = 0$

$\Rightarrow -\sin(\theta+\alpha)\sin\theta + \cos(\theta+\alpha)\cos\theta = 0$

Cos(A+B) formula $\Rightarrow \cos(\theta+\alpha+\theta) = 0$

$\Rightarrow \cos(2\theta+\alpha) = 0$

$\Rightarrow 2\theta+\alpha = \frac{\pi}{2} \Rightarrow 2\theta = \frac{\pi}{2} - \alpha \Rightarrow \boxed{\theta_{max} = \frac{1}{2}\left[\frac{\pi}{2} - \alpha\right]}$