

1975 :



$$u^2 = 216 \cos^2 \alpha + 218 \sin^2 \alpha$$

$$\vec{r}(t) = 216 \cos \alpha t \hat{i} + (218 \sin \alpha t - \frac{g}{2} t^2) \hat{j}$$

At a time t :



$$216 \cos \alpha t = 30 \quad (1)$$

$$218 \sin \alpha t - \frac{g}{2} t^2 = 10 \quad (2)$$

These are non-linear equations in α and t .
Solve either by :

$$218 \sin \alpha = \frac{10 + \frac{g}{2} t^2}{t} \quad \text{and} \quad 216 \cos \alpha = \frac{30}{t}$$

$$(218 \sin \alpha)^2 + (216 \cos \alpha)^2 = (21)^2 (\sin^2 \alpha + \cos^2 \alpha) = (21)^2 = 441$$

$$\Rightarrow \left(\frac{10 + \frac{g}{2} t^2}{t} \right)^2 + \left(\frac{30}{t} \right)^2 = (21)^2$$

$$\Rightarrow (10 + 4.4 t^2)^2 + (30)^2 = (21)^2 t^2 \quad \boxed{\text{solve for } t^2}$$

OR Better

$$(1) \quad t = \frac{30}{216 \cos \alpha}$$

$$\Rightarrow (2) \quad 218 \sin \alpha \left(\frac{30}{216 \cos \alpha} \right) - \frac{g}{2} \left(\frac{30}{216 \cos \alpha} \right)^2 = 10$$

$$\Rightarrow 30 \tan \alpha - \frac{4.9(900)}{(21)^2 \cos^2 \alpha} = 10$$

$$\Rightarrow 30 \tan \alpha - \frac{4410}{441 \cos^2 \alpha} = 10$$

$$\Rightarrow 30 \tan \alpha - 10 \sec^2 \alpha = 10$$

$$\Rightarrow 3 \tan \alpha - (1 + \tan^2 \alpha) = 1$$

$$\Rightarrow 3x - (1 + x^2) = 1$$

$$\Rightarrow 3x - 1 - x^2 = 1$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

$\Rightarrow x=2$ OR $x=1$
 $\Rightarrow \tan \alpha = 2$ and $\tan \alpha = 1$
 $\Rightarrow \alpha = 63.43^\circ$ and $\alpha = 45^\circ$

KEY IS

$$\boxed{\tan^2 \alpha = 1 + \sec^2 \alpha}$$

Quadratic for $\tan \alpha$.

$$\Rightarrow t = \frac{30}{216 \cos 63.43} = 3.194$$

OR $t = \frac{30}{216 \cos 45} = 2.02 \text{ secs.}$