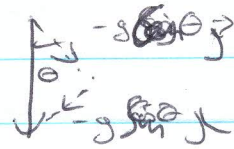
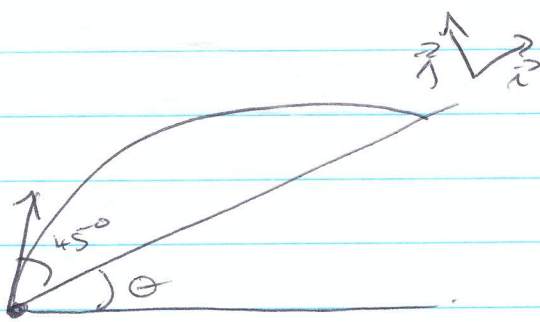


Projectile Motion

1974



$$\theta = \tan^{-1} \frac{1}{2}$$



$$u = \frac{u}{\sqrt{2}} \hat{i} + \frac{u}{\sqrt{2}} \hat{j}$$

$$(i) \quad \vec{r}(t) = \left(\frac{u}{\sqrt{2}} t - \frac{1}{2} g t^2 \cos^2 \theta \right) \hat{i} + \left(\frac{u}{\sqrt{2}} t - \frac{1}{2} g t^2 \sin^2 \theta \right) \hat{j} = \left(\frac{u}{\sqrt{2}} t - \frac{g}{2\sqrt{5}} t^2 \right) \hat{i} + \left(\frac{u}{\sqrt{2}} t - \frac{g}{2\sqrt{5}} t^2 \right) \hat{j}$$

$$\vec{v}(t) = \left(\frac{u}{\sqrt{2}} - g t \cos^2 \theta \right) \hat{i} + \left(\frac{u}{\sqrt{2}} - g t \sin^2 \theta \right) \hat{j}$$

$$\vec{v}(t) = \left(\frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}} t \right) \hat{i} + \left(\frac{u}{\sqrt{2}} - \frac{2g}{\sqrt{5}} t \right) \hat{j}$$

(ii) Particle strike plane at 90° .

Prove that $(\vec{v})_{\hat{j}}$ for $t = \text{time in flight} = 0$.

Time in flight: $(\hat{i})_{\hat{j}} = 0$.

$$\Rightarrow \frac{u}{\sqrt{2}} t - \frac{g}{\sqrt{5}} t^2 = 0$$

$$\Rightarrow t = 0 \text{ OR } t = \frac{u \sqrt{5}}{\sqrt{2} g}$$

$$\therefore (\vec{v})_{\hat{j}} \text{ for } t = \frac{u \sqrt{5}}{\sqrt{2} g} = \frac{u}{\sqrt{2}} - \frac{g}{\sqrt{5}} \left[\frac{u \sqrt{5}}{\sqrt{2} g} \right]$$

$$= \frac{u}{\sqrt{2}} - \frac{u}{\sqrt{2}}$$

$$= 0$$

(iii) Range = $(\vec{r})_{\hat{i}}$ for $t = \frac{u \sqrt{5}}{\sqrt{2} g}$

$$\Rightarrow \text{Range} = \frac{u}{\sqrt{2}} \left(\frac{u \sqrt{5}}{\sqrt{2} g} \right) - \frac{1}{2} g \frac{1}{\sqrt{5}} \left[\frac{u \sqrt{5}}{\sqrt{2} g} \right]^2$$

$$= \frac{u^2 \sqrt{5}}{g \cdot 2} - \frac{g}{2\sqrt{5}} \frac{[u^2 (5)]}{2 g^2}$$

$$= \frac{u^2 \sqrt{5}}{g \cdot 2} - \frac{u^2 \sqrt{5}}{4 g}$$

$$= \frac{u^2 \sqrt{5}}{g} \left(\frac{1}{2} - \frac{1}{4} \right)$$

$$= \frac{u^2 \sqrt{5}}{g} \frac{1}{4}$$