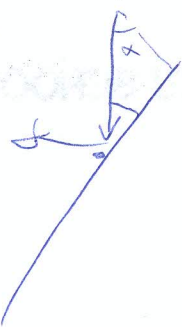


1973Q2

$$e=1$$

Collision:

$$\vec{u} = u \sin \alpha \vec{i} - u \cos \alpha \vec{j}$$

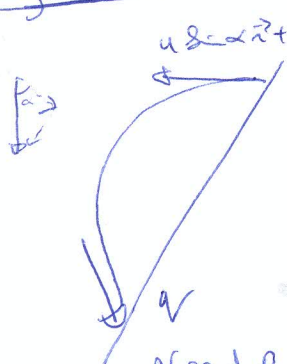
$$\vec{v} = u \sin \alpha \vec{i} + y \vec{j} \quad (\text{Smoothness} \Rightarrow \vec{i} \text{ unchanged})$$

$$e=1: v_2 - v_1 = -e(u_2 - u_1)$$

$$\Rightarrow y - 0 = -1(-u \cos \alpha - 0)$$

$$\Rightarrow y = u \cos \alpha$$

$$\therefore \vec{v} = u \sin \alpha \vec{i} + u \cos \alpha \vec{j}$$

Projectile

$$u \sin \alpha \vec{i} + u \cos \alpha \vec{j} \quad " \vec{u} " = u \sin \alpha \vec{i} + u \cos \alpha \vec{j}$$

$$\vec{g} = g \sin \alpha \vec{i} - g \cos \alpha \vec{j}$$

$$\vec{v}(t) = \left(u \sin \alpha + g \sin \alpha t \right) \vec{i} + \left(u \cos \alpha - g \cos \alpha t \right) \vec{j}$$

$$\vec{r}(t) = \left(u \sin \alpha t + \frac{g \sin \alpha t^2}{2} \right) \vec{i} + \left(u \cos \alpha t - \frac{g \cos \alpha t^2}{2} \right) \vec{j}$$

Need Range : Need T :

$$\left(\vec{r}(t) \right)_{\vec{j}} = 0 \Rightarrow u \cos \alpha t - g \frac{\cos \alpha}{2} t^2 = 0$$

$$\Rightarrow t = 0 \text{ OR } t = \frac{2u \cos \alpha}{g \cos \alpha} = \frac{2u}{g}$$

$$\begin{aligned} \therefore \text{Range} &= u \sin \alpha \left(\frac{2u}{g} \right) + \frac{g \sin \alpha}{2} \left(\frac{2u}{g} \right)^2 \\ &= \frac{2u^2 \sin \alpha}{g} + \frac{2u^2 \sin \alpha}{g} \\ &= \frac{4u^2 \sin \alpha}{g} \end{aligned}$$

Speed at y before strikes:

$$\vec{v} \left(\frac{2u}{g} \right) = \left(u \sin \alpha + g \sin \alpha \left(\frac{2u}{g} \right) \right) \vec{i} + \left(u \cos \alpha - g \cos \alpha \left(\frac{2u}{g} \right) \right) \vec{j}$$

$$\vec{v} = (3u \sin \alpha) \vec{i} - (u \cos \alpha) \vec{j}$$

Collision (2)

$$\vec{v}_{\text{before}} = 3u \sin \alpha \vec{i} - u \cos \alpha \vec{j}$$

$$\vec{v}_{\text{after}} = 3u \sin \alpha \vec{i} + u \cos \alpha \vec{j} \quad (\text{Smoothness and NLR } e=1 \text{ in } \vec{j} \text{ dir})$$

$$\text{Proj: } \vec{r}(t) = \left(3u \sin \alpha t + \frac{g \sin \alpha t^2}{2} \right) \vec{i} + \left(u \cos \alpha t + \frac{g \cos \alpha t^2}{2} \right) \vec{j}$$

$$\Rightarrow \left(\vec{r}(t) \right)_{\vec{j}} = 0 \Rightarrow t = 0 \text{ OR } t = \frac{2u}{g}$$

$$\text{Range} = \left(\vec{r} \left(\frac{2u}{g} \right) \right)_{\vec{i}} = 3u \sin \alpha \left(\frac{2u}{g} \right) + \frac{g \sin \alpha}{2} \left(\frac{2u}{g} \right)^2 = \frac{8u^2 \sin \alpha}{g}$$