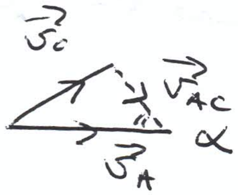
1983 Q2

$P \cdot$ $\vec{j}$ $\vec{i}$ $\cdot Q$

Outward

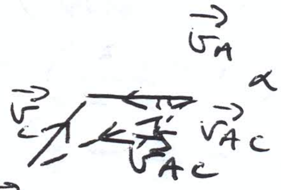


$\vec{v}_A = u_1 \vec{i}$

$\vec{v}_C = \frac{v}{\sqrt{2}}\vec{i} + \frac{v}{\sqrt{2}}\vec{j}$

$\vec{v}_{AC} = x\cos\alpha\,\vec{i} - x\sin\alpha\,\vec{j}$

Return



$\vec{v}_A = -u_2 \vec{i}$ (N.B. the minus)

$\vec{v}_C = \frac{v}{\sqrt{2}}\vec{i} + \frac{v}{\sqrt{2}}\vec{j}$

$\vec{v}_{AC} = -x\cos\alpha\,\vec{i} - x\sin\alpha\,\vec{j}$.

In **BOTH** cases

$$\vec{v}_A = \vec{v}_{AC} + \vec{v}_C$$

$\Rightarrow u_1\vec{i} = x\cos\alpha\,\vec{i} - x\sin\alpha\,\vec{j} + \frac{v}{\sqrt{2}}\vec{i} + \frac{v}{\sqrt{2}}\vec{j}$

$\vec{i}$ cpts: $u_1 = x\cos\alpha + \frac{v}{\sqrt{2}}$ (1)

$\vec{j}$ cpts: $0 = -x\sin\alpha + \frac{v}{\sqrt{2}}$ (2)

$\Rightarrow -u_2\vec{i} = -x\cos\alpha\,\vec{i} - x\sin\alpha\,\vec{j} + \frac{v}{\sqrt{2}}\vec{i} + \frac{v}{\sqrt{2}}\vec{j}$

$\vec{i}$ cpts $-u_2 = -x\cos\alpha + \frac{v}{\sqrt{2}}$ (*)

$\vec{j}$ cpts $0 = -x\sin\alpha + \frac{v}{\sqrt{2}}$ (**)

We now have to use (1) (2) (*) and (**) to find what we were asked!

<u>Note</u>: (2) and (**) tell us that angle headed off in is definitely the same on both parts of the journey <u>and</u> that $\boxed{\sin\alpha = \frac{v}{x\sqrt{2}}}$ (H)

Next to show $u_1 - u_2 = v\sqrt{2}$ and $u_1 u_2 = x^2 - v^2$ we just use (1) and (*)

1st <u>Add</u> (1) and (*) $\Rightarrow$ $u_1 + (-u_2) = x\cos\alpha + \frac{v}{\sqrt{2}} - x\cos\alpha + \frac{v}{\sqrt{2}}$

$\Rightarrow$ $u_1 - u_2 = 2\frac{v}{\sqrt{2}}$

$\Rightarrow$ $u_1 - u_2 = \sqrt{2}\,v$ qed.

Second:

$u_1 = x\cos\alpha + \frac{v}{\sqrt{2}}$ $\quad u_2 = x\cos\alpha - \frac{v}{\sqrt{2}}$

$\Rightarrow u_1 \cdot u_2 = \left(x\cos\alpha + \frac{v}{\sqrt{2}}\right)\left(x\cos\alpha - \frac{v}{\sqrt{2}}\right)$

$= (x\cos\alpha)^2 - \left(\frac{v}{\sqrt{2}}\right)^2$ (because $(a+b)(a-b) = a^2 - b^2$.