

1483, Q2 ctd

$$\Rightarrow u_1 - u_2 = x^2 \cos^2 \alpha - \frac{v^2}{2}$$

$$\Rightarrow u_1 - u_2 = x^2 (1 - \sin^2 \alpha) - \frac{v^2}{2} \quad (\text{Because } \cos^2 \alpha + \sin^2 \alpha = 1)$$

$$\Rightarrow u_1 - u_2 = x^2 \left(1 - \left(\frac{v}{x\sqrt{2}}\right)^2\right) - \frac{v^2}{2} \quad (\text{Because of Equ (H)})$$

$$\Rightarrow u_1 - u_2 = x^2 \left(1 - \frac{v^2}{x^2 2}\right) - \frac{v^2}{2}$$

$$\Rightarrow u_1 - u_2 = x^2 - \frac{v^2}{2} - \frac{v^2}{2}$$

$$\Rightarrow u_1 - u_2 = x^2 - v^2 \quad \text{qed (!)}$$

Expression for Time for return journey: $T = \frac{d}{u_1} + \frac{d}{u_2}$ Speed u_1 and u_2

We could do either of the three methods that follow

WAY ONE: (SLOP)
Find u_1, u_2 by solving

$$u_1 - u_2 = v\sqrt{2} \quad \text{(A)}$$

$$u_1 u_2 = x^2 - v^2 \quad \text{(B)}$$

Non linear simultaneous equations in (A) and (B)

$$\text{(A)} \Rightarrow u_1 = v\sqrt{2} + u_2$$

$$\text{(B)} \Rightarrow$$

$$(v\sqrt{2} + u_2)u_2 = x^2 - v^2$$

$$\Rightarrow v\sqrt{2}u_2 + u_2^2 = x^2 - v^2$$

$$\Rightarrow u_2^2 + v\sqrt{2}u_2 - (x^2 - v^2) = 0$$

Quadratic in u_2 with
 $a = 1$ $b = v\sqrt{2}$ $c = -x^2 + v^2$

$$u_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$u_2 = \frac{-v\sqrt{2} \pm \sqrt{(v\sqrt{2})^2 - 4(x^2 - v^2)}}{2}$$

$$u_2 = \frac{-v\sqrt{2} \pm \sqrt{2v^2 - 4x^2 + 4v^2}}{2}$$

$$u_2 = \frac{-v\sqrt{2} \pm \sqrt{4x^2 - 2v^2}}{2}$$

As u_2 is supposed to be +ve that the top one

$$u_2 = \frac{-v\sqrt{2} + \sqrt{4x^2 - 2v^2}}{2}$$

$$\therefore u_1 = v\sqrt{2} + \frac{-v\sqrt{2} + \sqrt{4x^2 - 2v^2}}{2}$$

$$\Rightarrow u_1 = \frac{v\sqrt{2} + \sqrt{4x^2 - 2v^2}}{2}$$

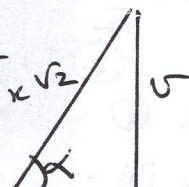
$$\text{And } T = \frac{d}{u_1} + \frac{d}{u_2}$$

with these subbed in

WAY TWO: (TRIG)

$$\text{If } \sin \alpha = \frac{v}{x\sqrt{2}}$$

$$\cos \alpha = \frac{\text{adj}}{x\sqrt{2}}$$



adjacent

$$v^2 + \text{adj}^2 = (x\sqrt{2})^2$$

$$\Rightarrow \text{adj}^2 = 2x^2 - v^2$$

$$\text{adj} = \sqrt{2x^2 - v^2}$$

$$\Rightarrow \cos \alpha = \frac{\sqrt{2x^2 - v^2}}{x\sqrt{2}}$$

$$\Rightarrow u_1 = x \cos \alpha + \frac{v}{\sqrt{2}}$$

$$u_1 = x \frac{\sqrt{2x^2 - v^2}}{x\sqrt{2}} + \frac{v}{\sqrt{2}}$$

$$u_1 = \frac{\sqrt{2x^2 - v^2}}{\sqrt{2}} + \frac{v}{\sqrt{2}}$$

(as above)

and

$$u_2 = +x \cos \alpha - \frac{v}{\sqrt{2}}$$

$$= x \frac{\sqrt{2v^2 - x^2}}{x\sqrt{2}} - \frac{v}{\sqrt{2}}$$

$$u_2 = -\frac{v}{\sqrt{2}} + \frac{\sqrt{2v^2 - x^2}}{\sqrt{2}}$$

(as above)

and again

$$T = \frac{d}{u_1} + \frac{d}{u_2} \text{ with these values in!}$$

WAY THREE: (GENIUS!)

$$T = \frac{d}{u_1} + \frac{d}{u_2}$$

$$\Rightarrow T = d \left(\frac{1}{u_1} + \frac{1}{u_2} \right)$$

$$\Rightarrow T = d \left(\frac{u_2 + u_1}{u_1 u_2} \right)$$

$$\Rightarrow T^2 = d^2 \frac{(u_1 + u_2)^2}{(u_1 u_2)^2}$$

$$\text{But } (u_1 + u_2)^2 = (u_1 - u_2)^2 + 4u_1 u_2$$

$$\Rightarrow (u_1 + u_2)^2 = (v\sqrt{2})^2 + 4(x^2 - v^2)$$

$$\Rightarrow (u_1 + u_2)^2 = 2v^2 + 4x^2 - 4v^2 = 4x^2 - 2v^2$$

$$\Rightarrow T^2 = \frac{d^2 (4x^2 - 2v^2)}{(x^2 - v^2)^2}$$

$$\Rightarrow T = \frac{d \sqrt{4x^2 - 2v^2}}{x^2 - v^2}$$