Q10 (D)

(i)

$\text{Force} = -mkv^2$

$ma = -mkv^2$

$\cancel{\frac{dv}{dt}} = -kv^2$

$a = -kv^2$

$\Rightarrow \frac{dv}{dt} = -kv^2$ or $v\frac{dv}{dx} = -kv^2$

See typed alternative (ii) (5)

$x = 0$, $t = 0$, $v = 14$ ——— $x = ?$, $t = 10$, $v = 7$

Link v and t: $\frac{dv}{dt} = -kv^2$ (5)

$\Rightarrow \int_{14}^{7} \frac{1}{v^2}\,dv = \int_0^{10} -k\,dt$

$\int_{14}^{7} v^{-2}\,dv = -kt\Big|_0^{10}$

$-\frac{1}{v}\Big|_{14}^{7} = -k(10) + k(0)$ (5)

$-\left[\frac{1}{7} - \frac{1}{14}\right] = -k10$

$\frac{1}{140} = k$ (5)

(ii) method A (15)

$x = 0$, $v = 14$, $t = 0$ ——— $x = s$, $v = v$, $t = T$

$\Rightarrow \int_{14}^{v} v^{-2}\,dv = \int_0^t -k\,dt$

$\Rightarrow -\frac{1}{v}\Big|_{14}^{v} = -kt\Big|_0^t$

$\Rightarrow -\left[\frac{1}{v} - \frac{1}{14}\right] = -\frac{1}{140}[t-0]$

$\Rightarrow \frac{1}{v} = \frac{10}{140} + \frac{t}{140}$

$\Rightarrow \frac{1}{v} = \frac{10+t}{140}$

$\Rightarrow v = \frac{140}{10+t}$

$\therefore \frac{dx}{dt} = \frac{140}{10+t}$

$\int_0^s dx = 140\int_0^T \frac{1}{10+t}\,dt$

$x\Big|_0^s = 140\ln(10+t)\Big|_0^T$

$s - 0 = 140(\ln(10+T) - \ln(10))$

$s = 140\ln\left(\frac{10+T}{10}\right)$