1996 HLC APM

Q10

$$\frac{dy}{dx} = 4y\cos x \quad , y = e^2, x = \frac{\pi}{6}$$

$$\int \frac{dy}{y} = \int 4\cos x \, dx \qquad \boxed{5}$$

$$\ln y = 4\sin x + C \qquad \boxed{5}$$

// $y = e^2$, $x = \frac{\pi}{6} \Rightarrow$

$$\ln e^2 = 4\sin\frac{\pi}{6} + C$$

$$2 = 4\left(\tfrac{1}{2}\right) + C$$

$$0 = C \ //$$

$$\Rightarrow \ln y = 4\sin x + 0$$

$$\Rightarrow \ln y = 4\sin x \qquad \boxed{5}$$

$$\Rightarrow e^{\ln y} = e^{4\sin x}$$

$$\Rightarrow \boxed{y = e^{4\sin x}} \qquad \boxed{5}$$

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air resistance $= kv^2$ per unit mass.

Forces:

mkv^2 , mg

SIGNS $\uparrow +$

NII: $\Sigma F = ma$

$$-mkv^2 - mg = ma$$

$$\boxed{-kv^2 - g = a}$$

(i) greatest height:

$v = 0$, $x = H$, $(t = T)$

$t = 0$
$x = 0$
$v = \sqrt{\frac{2g}{k}}$

Link v and x: $v\frac{dv}{dx} = -kv^2 - g$

$$\int_{\sqrt{\frac{2g}{k}}}^{0} \frac{v\,dv}{kv^2 + g} = \int_0^H -1 \, dx$$

$$= -x \Big|_0^H$$

// $u = kv^2 + g \Rightarrow \frac{du}{dv} = k2v$

$$\Rightarrow \frac{1}{2k}du = v\,dv$$

$$\int \frac{v\,dv}{kv^2 + g} = \int \frac{\frac{1}{2k}du}{u}$$

$$= \frac{1}{2k}\ln u$$

$$= \frac{1}{2k}\ln(kv^2 + g) \ //$$

$$\Rightarrow \frac{1}{2k}\ln(kv^2 + g)\Big|_{\sqrt{\frac{2g}{k}}}^{0} = -x\Big|_0^H$$

$$\Rightarrow \frac{1}{2k}\left[\ln(g) - \ln\left(k\left(\frac{2g}{k}\right) + g\right)\right] = -H \qquad \boxed{10}$$

$$\Rightarrow \frac{1}{2k}\left[\ln 3g - \ln g\right] = H$$

$$\Rightarrow \frac{1}{2k}\ln\left(\frac{3g}{g}\right) = H$$

$$\Rightarrow \boxed{H = \frac{\ln 3}{2k}} \qquad \boxed{5}$$

(ii) on way back down: $\downarrow \oplus \ \uparrow \ominus$

$-mkv^2$, mg

$$\Sigma F = ma$$

$$mg - mkv^2 = ma$$

$$g - kv^2 = a$$

$x = 0 \quad v = 0$

$x = \frac{\ln 3}{2k} \quad v = V$

$$v\frac{dv}{dx} = g - kv^2$$

$$\Rightarrow \int_0^V \frac{v}{g - kv^2}\,dv = \int_0^{\frac{\ln 3}{2k}} 1 \, dx$$

// $\int \frac{v}{g - kv^2}dv \Rightarrow u = g - kv^2 \Rightarrow \frac{du}{dv} = -2kv$

$$\Rightarrow -\frac{1}{2k}du = v\,dv$$

$$\Rightarrow \int \frac{v}{g - kv^2}dv = \int \frac{-\frac{1}{2k}}{u}du = -\frac{1}{2k}\ln u \ //$$

$$\Rightarrow -\frac{1}{2k}\ln(g - kv^2)\Big|_0^V = x\Big|_0^{\frac{\ln 3}{2k}} \qquad \boxed{10}$$

$$\Rightarrow -\frac{1}{2k}\ln(g - kV^2) + \frac{1}{2k}\ln g = \frac{\ln 3}{2k} - 0$$

$$\Rightarrow \ln\left(\frac{g}{g - kV^2}\right) = \ln 3$$

$$\Rightarrow g = 3g - 3kV^2$$

$$\Rightarrow -2g = -3kV^2$$

$$\Rightarrow V = \sqrt{\frac{2g}{3k}} \quad \text{qed.} \qquad \boxed{5}$$