

1993 Q10

$$\frac{dy}{dx} = 28 - 2x + 604x$$

$$\int dy = \int (28 - 2x + 604x) dx + C$$

$$\Rightarrow y = -2 \frac{602x}{2} + \frac{28 \cdot 4x}{4} + C$$

$$y = -602x + \frac{1}{4} 28 \cdot 4x + C$$

// $x = \frac{\pi}{4}$ when $y = 1$

$$\Rightarrow 1 = -602\left(\frac{\pi}{4}\right) + \frac{1}{4} 28 \cdot \left(\frac{\pi}{4}\right) + C$$

$$\Rightarrow 1 = 0 + \frac{1}{4} 28 \cdot \pi + C$$

$$\Rightarrow 1 = 0 + 0 + C$$

$$\Rightarrow 1 = C$$

$$\Rightarrow y = 602x + \frac{1}{4} 28 \cdot 4x + 1$$

At $x = \frac{\pi}{2}$

$$\Rightarrow y = -602\left(\frac{\pi}{2}\right) + \frac{1}{4} 28 \cdot \left(\frac{\pi}{2}\right) + 1$$

$$\Rightarrow y = -60\pi + \frac{1}{4} 28 \cdot 2\pi + 1$$

$$\Rightarrow y = +1 + 0 + 1$$

$$\Rightarrow y = 2$$

$$\Rightarrow \ln 1 - \ln 3 = -\frac{\pi}{2}$$

$$\Rightarrow -\ln 3 = -\frac{\pi}{2}$$

$$\Rightarrow 2 \ln 3 = \pi$$

$$\Rightarrow \pi = \ln 3^2 = \ln 9$$

Reduce an expression for v at any time T .

$x=0$	$x=X$
$v=210$	$v=v$
$t=0$	$t=T$

$$\Rightarrow \int \frac{dv}{v} \frac{dv}{dt} = -kv$$

$$\Rightarrow \int_{210}^v \frac{dv}{v} = \int_0^T -k dt$$

$$\Rightarrow \ln v \Big|_{210}^v = -k t \Big|_0^T$$

$$\Rightarrow \ln v - \ln 210 = -(\ln 9) T + 0$$

$$\Rightarrow \ln \frac{v}{210} = -(\ln 9) T$$

$$\Rightarrow \frac{v}{210} = e^{-(\ln 9) T}$$

$$\Rightarrow v = 210 e^{-(\ln 9) T} \quad (\text{ASK})$$

(f) accel = $-kv$

$v=210$	$v=70$
$t=0$	$t=0.5$
$x=0$	$x=X$

$$\frac{dv}{dt} = -kv$$

$$\Rightarrow \int_{210}^{70} \frac{dv}{v} = \int_0^{0.5} -k dt$$

$$\Rightarrow \ln v \Big|_{210}^{70} = -k t \Big|_0^{0.5}$$

$$\Rightarrow \ln 70 - \ln 210 = -k(0.5) + 0$$

$$\Rightarrow \ln \left(\frac{70}{210}\right) = -\frac{k}{2}$$

$$\Rightarrow \ln \left(\frac{1}{3}\right) = -\frac{k}{2}$$

(ii) Calculate x :

$x=0$	$x=X$
$v=210$	$v=70$
$t=0$	$t=0.5$

$$\text{accel} = -kv$$

$$v \frac{dv}{dx} = -kv$$

$$\Rightarrow \int_{210}^{70} \frac{dv}{v} = \int_0^X -k dx$$

$$= -kx$$

$$\Rightarrow 70 - 210 = -kx$$

$$\Rightarrow -140 = -(\ln 9)x$$

$$\Rightarrow 140 = \ln 9 x$$

$$\Rightarrow x = \frac{140}{\ln 9} = \frac{140}{\ln 3^2}$$

$$\Rightarrow x = \frac{140}{2 \ln 3} = \frac{70}{\ln 3}$$

$$\left[\text{Footnote } 210(e^{-(\ln 9)T}) = 210 e^{\ln \frac{1}{9} T} = 210 \left(e^{\ln \frac{1}{9}}\right)^T = 210 \left(\frac{1}{9}\right)^T = \frac{210}{9^T} \right]$$