1990 Q10 DE

$$x\frac{dy}{dx} = y(1+y)$$

$$\Rightarrow \int \frac{dy}{y+y^2} = \int \frac{dx}{x}$$

LHS $\int \frac{dy}{y+y^2}$

Either $y^2+y = y^2+y+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$= \left(y+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2.$

Let $u = y+\frac{1}{2} \Rightarrow \frac{du}{dy} = 1$

$\Rightarrow du = dy$

$$\Rightarrow \int \frac{dy}{y^2+y} = \int \frac{du}{u^2-\left(\frac{1}{2}\right)^2}$$

$$= -\int \frac{du}{\left(\frac{1}{2}\right)^2-u^2}$$

$$\cancel{= \int \frac{du}{u^2}}$$

$$= \frac{-1}{2\left(\frac{1}{2}\right)} \ln\left|\frac{\frac{1}{2}+u}{\frac{1}{2}-u}\right|$$

$$= -\ln\left|\frac{\frac{1}{2}+y+\frac{1}{2}}{\frac{1}{2}-\left(y+\frac{1}{2}\right)}\right|$$

$$= -\ln\left|\frac{y+1}{-y}\right|$$

$$= -\ln\left|\frac{y+1}{y}\right|$$

$$= \ln\left(\frac{y}{y+1}\right) \quad //$$

OR $\int \frac{dy}{y^2+y} = \int \frac{dy}{y(y+1)}$

$$= \int\left(\frac{1}{y}-\frac{1}{y+1}\right)dy$$

$$= \ln y - \ln(y+1).$$

$$= \ln\left(\frac{y}{y+1}\right) \quad //$$

$\therefore \ln\left(\frac{y}{y+1}\right) = \ln x + C$

ICs $\Rightarrow$. $x=1, y=1 \Rightarrow \ln\left(\frac{1}{2}\right) = \ln 1 + C$

$\Rightarrow \ln\frac{1}{2} = C \quad //$

$\Rightarrow \ln\left(\frac{y}{y+1}\right) = \ln x + \ln\left(\frac{1}{2}\right)$

$\Rightarrow \ln\left(\frac{y}{y+1}\right) = \ln\left(\frac{x}{2}\right).$

$\Rightarrow \frac{y}{y+1} = \frac{x}{2}$

$\Rightarrow 2y = xy + x$

$\Rightarrow 2y - xy = x$

$\Rightarrow y(2-x) = x$

$\Rightarrow y = \frac{x}{2-x}.$

(b) Forces changes 0sec to 10sec

(i) from 0N to 16N

$\Rightarrow$. At any time t. the force is

$$F(t) = \left(\frac{16-0}{10}\right)t$$

$$\left[\begin{matrix} 0 & 1 & 2 & \cdots & 10 \\ 0 & 1\cdot 6 & 3\cdot 2 & \cdots & 16 \end{matrix}\right]$$

$$\boxed{F(t) = 1\cdot 6t}$$

NII : $\sum \vec{F} = m\vec{a}$.

$\Rightarrow$. $1\cdot 6t = 8a$

$\Rightarrow \boxed{\cdot 2t = a}$ (E)

(ii) (E) $\Rightarrow \frac{dv}{dt} = \cdot 2t$.

$\Rightarrow \int dv = \int (\cdot 2t)\,dt$

$\Rightarrow v = \cdot 2\frac{t^2}{2} + C$

// $\Rightarrow v=0, t=0 \Rightarrow C=0$ //

$\Rightarrow \boxed{v = \cdot 1t^2}$ (1)

$\Rightarrow \frac{dx}{dt} = \cdot 1t^2$

$\Rightarrow \int dx = \int \cdot 1t^2\,dt$

$\Rightarrow x = \cdot 1\frac{t^3}{3} + C$

// $\Rightarrow x=0, t=0 \Rightarrow C=0$ //

$\Rightarrow x = \frac{1}{30}t^3.$

$\Rightarrow 3x = \frac{1}{10}t^3$

$\Rightarrow 9x^2 = \frac{1}{100}t^6$

(1) $\Rightarrow 9x^2 = \frac{1}{100}(10v)^3$

$\Rightarrow \boxed{9x^2 = 10v^3.}$ qed.