1987 (10) : N/3

(a) $2x(1+y)\frac{dx}{dy} = 8+x^2$ [5]

$\Rightarrow \int \frac{2x}{x^2+8}\,dx = \int \frac{1}{1+y}\,dy + A$ [5]

$\Rightarrow \ln(x^2+8) = \ln(1+y) + A$

// $x=2, y=3 \Rightarrow$

$\Rightarrow \ln(2^2+8) = \ln(4) + A$

$\Rightarrow \ln 12 = \ln 4 + A$ [5]

$\Rightarrow \ln 12 - \ln 4 = A$

$\Rightarrow \ln\left(\frac{12}{4}\right) = A$

$\Rightarrow \ln 3 = A$ //

$\Rightarrow \ln(x^2+8) = \ln(1+y) + \ln 3$

$\Rightarrow \ln(x^2+8) = \ln[3(1+y)]$

$\Rightarrow x^2+8 = 3(1+y)$

$\Rightarrow \boxed{x^2 = 3y-5}$.

$\Rightarrow \boxed{x = \sqrt{3y-5}}$ [5]

(b) on the level : we need to calculate the power output of the train as this is we are told the power output on the slope also.

Forces: R, T

Speed = Constant $= 16\,ms^{-1}$

R = 60w per tonne

NII $\Rightarrow$ $T - R = 0$. $[a=0]$

$\Rightarrow T = R$

[5] $\Rightarrow T = \frac{60\,m}{1000}$ [careful of units]

Power output $= T.v$

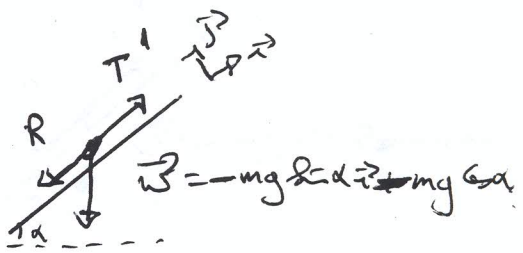
$= \left(\frac{60m}{1000}\right)16$.

$= \frac{96m}{100}$.

On the slope :

Power output of engine $= \frac{96m}{100}$

Forces:



Along plane :

NII $\Rightarrow$ $ma = \Sigma F$.

$\Rightarrow \boxed{m\frac{dv}{dt} = T' - mg\sin\alpha - R}$ (*)

BUT : Power $= \frac{96m}{100}$

$\Rightarrow T'v = \frac{96m}{100}$ $\Rightarrow \boxed{T' = \frac{96m}{100v}}$ [5]

(*) $\Rightarrow$

$m\frac{dv}{dt} = \frac{96m}{100v} - \frac{mg}{98} - \frac{60m}{1000}$ [5]

$\Rightarrow \frac{dv}{dt} = \frac{96}{100v} - \frac{9.8}{98} - \frac{6}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{96}{100v} - \frac{1}{10} - \frac{6}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{96}{100v} - \frac{16}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{16}{100}\left(\frac{6}{v} - 1\right)$

$\Rightarrow \boxed{\frac{dv}{dt} = \frac{4}{25}\left(\frac{6-v}{v}\right)}$ [5]

$\Rightarrow \int_{16}^{12} \frac{v\,dv}{6-v} = \int_0^T \frac{4}{25}\,dt$

$\Rightarrow \int_{16}^{12} \frac{v}{v-6}\,dv = \int_0^T -\frac{4}{25}\,dt$

// $\int \frac{v}{v-6}\,dv = \int \frac{v+6-6}{v-6}\,dv$

$= \int \left(\frac{v-6}{v-6} + \frac{6}{v-6}\right)dv$

Let $u = v-6$.

$\Rightarrow u+6 = v$

$du = dv$

$\Rightarrow \int \frac{v}{v-6}\,dv = \int \frac{u+6}{u}\,du$

$= \int 1 + \frac{6}{u}\,du$

$= [u + 6\ln u]$

$= [(v-6) + 6\ln(v-6)]$ //