1484Q10 (a) $\frac{dv}{dt} = g - kv \Rightarrow \int \frac{dv}{g-kv} = \int dt + C$

$\boxed{\begin{array}{l} \text{Let } u = g - kv \\ \frac{du}{dv} = -k \\ -\frac{1}{k}du = dv \end{array}}$

$\Rightarrow -\frac{1}{k}\ln(g-kv) = t + C$

$\Rightarrow \ln(g-kv) = -kt - kC$

$\Rightarrow \ln(g-kv) = -kt + D$ [call $D \equiv -kC$]

$\Rightarrow g - kv = e^{(-kt+D)}$

$\Rightarrow -kv = -g + e^{(-kt+D)}$

$\Rightarrow v = \frac{1}{k}\left(+g - e^{(-kt+D)}\right)$

$v = \frac{1}{k}\left[g - e^{(-kt+D)}\right]$

As $t \to \infty$ $e^{-kt+D} \to 0$

$\Rightarrow v \to \frac{g}{k}$ qed

(b) Signs: $+$ → makes life easier - ask.

Forces: ← $-mkv^2$

Conditions at positions: P ↔ Q, 100

	P	Q
	$v = 20$	$v = 10$
Let	$x = 0$	$x = 100$
Let	$t = 0$	$t = T$

NII: $\Rightarrow m(\text{accel}) = -mkv^2$

$\Rightarrow$ accel $= -kv^2$

Link v and t:

$\frac{dv}{dt} = -kv^2$

$\Rightarrow \int_{20}^{10} \frac{-dv}{v^2} = \int_0^T k\,dt$

$\Rightarrow -\left(\frac{-1}{v}\right)\Big|_{20}^{10} = kt\Big|_0^T$

$\Rightarrow \frac{1}{v}\Big|_{20}^{10} = kT$

$\Rightarrow \frac{1}{10} - \frac{1}{20} = kT.$

$\Rightarrow +\frac{1}{20} = kT$

Need k

Link v and x (To find k)

$v\frac{dv}{dx} = -kv^2$

$\Rightarrow \int_{20}^{10} \frac{dv}{v} = \int_0^{100} -k\,dx$

$\Rightarrow \ln v\Big|_{20}^{10} = -kx\Big|_0^{100}$

$\Rightarrow \ln 10 - \ln 20 = -k100 + 0$

$\Rightarrow \ln\frac{10}{20} = -k100$

$\Rightarrow \ln\frac{1}{2} = -k100$

$\Rightarrow \ln(2^{-1}) = -k100$

$\Rightarrow -\ln 2 = -k100$

$\Rightarrow \frac{\ln 2}{100} = k$

$\therefore kT = \frac{1}{20} \Rightarrow \frac{\ln 2}{100}T = \frac{1}{20} \Rightarrow \boxed{T = \frac{5}{\ln 2}.}$