DE 1982

Q1(a) $(1+x^3)\dfrac{dy}{dx} = x^2 y$

$\Rightarrow \quad \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{x^2}{1+x^3}$

$\Rightarrow \quad \displaystyle\int \frac{dy}{y} = \int \frac{x^2}{1+x^3}\,dx + C \quad (*)$

Substitution: let $u = 1+x^3$

$\Rightarrow du = 3x^2\,dx$

$\Rightarrow \frac{1}{3}du = x^2\,dx$

$\therefore \displaystyle\int \frac{x^2}{1+x^3}\,dx = \int \frac{1}{3}\frac{1}{u}\,du = \frac{1}{3}\ln u$

$= \frac{1}{3}\ln(1+x^3)$

$\therefore (*) \Rightarrow \ln y = \frac{1}{3}\ln(1+x^3) + C$

$y=2, x=1 \Rightarrow \ln 2 = \frac{1}{3}\ln(1+1) + C$

$\Rightarrow \ln 2 = \frac{1}{3}\ln 2 + C$

$\Rightarrow C = \frac{2}{3}\ln 2$

$\therefore \ln y = \frac{1}{3}\ln(1+x^3) + \frac{2}{3}\ln 2$

$\ln y = \frac{1}{3}[\ln(1+x^3) + 2\ln 2]$

$= \frac{1}{3}[\ln(1+x^3) + \ln 2^2]$

$= \frac{1}{3}[\ln[(1+x^3)\cdot 4]]$

$3\ln y = \ln 4(1+x^3)$

$\ln y^3 = \ln 4(1+x^3)$

Now Remove ln $\Rightarrow \boxed{y^3 = 4(1+x^3)}$

(b) $\dfrac{d^2s}{dt^2} = -\left(\dfrac{ds}{dt}\right)^2 \quad$ Let $v = \dfrac{ds}{dt}$

$\Rightarrow \dfrac{dv}{dt} = -v^2$

$\Rightarrow -\dfrac{1}{v^2}\,dv = dt$

$\Rightarrow \displaystyle\int -\frac{1}{v^2}\,dv = \int dt + A$

$\Rightarrow -\left(-\dfrac{1}{v}\right) = t + A$

WATCH SIGNS! $\Rightarrow \dfrac{1}{v} = t + A$

$t=0$, when $v=1 \Rightarrow A = 1$

$\dfrac{1}{v} = t+1$

$\Rightarrow \boxed{v = \dfrac{1}{t+1}}$

$\Rightarrow \dfrac{ds}{dt} = \dfrac{1}{t+1}$

$\Rightarrow \displaystyle\int ds = \int \frac{dt}{t+1} + B$

$\Rightarrow s = \ln(t+1) + B$

$s=0, t=0 \Rightarrow$

$0 = \ln(1) + B$

$\Rightarrow B = 0$

$\therefore \boxed{s = \ln(t+1)}$

The particle movement is described by the equation and the conditions in (b) exactly so the solution we have established to the DE applies

$s = \ln(t+1)$

for $t=1$ $\quad s = \ln(1+1)$

$\boxed{s = \ln 2}$