

1981

Q10(b):

$$\frac{d^2y}{dx^2} = k \frac{dy}{dx}$$

$$\Rightarrow \frac{d\left(\frac{dy}{dx}\right)}{dx} = k \frac{dy}{dx}$$

$$\Rightarrow \frac{dp}{dx} = kp$$

$$\Rightarrow \int \frac{dp}{p} = k \int dx$$

$$\Rightarrow \ln p = kx + c$$

$$\Rightarrow p = e^{(kx+c)}$$

$$\Rightarrow \frac{dy}{dx} = e^{(kx+c)}$$

$$\Rightarrow \int dy = \int e^{(kx+c)} dx$$

$$\Rightarrow y = \frac{1}{k} e^{(kx+c)} + D.$$

Sign: $x \rightarrow$ 

$$\frac{dx}{dt^2} = \frac{1}{2} \frac{dx}{dt}$$

$$v = \frac{dx}{dt} \Rightarrow \frac{dv}{dt} = \frac{1}{2} v$$

$$\Rightarrow \int \frac{dv}{v} = \int \frac{1}{2} dt + c$$

$$\Rightarrow \ln v = \frac{t}{2} + c$$

$$t=0, v=3 \Rightarrow \ln 3 = \frac{0}{2} + c$$

$$\Rightarrow \ln v = \frac{t}{2} + \ln 3$$

$$\Rightarrow \ln v - \ln 3 = \frac{t}{2}$$

$$\Rightarrow \ln\left(\frac{v}{3}\right) = \frac{t}{2}$$

$$\Rightarrow \frac{v}{3} = e^{\frac{t}{2}}$$

$$\Rightarrow v = 3e^{\frac{t}{2}}$$

$$\Rightarrow \frac{dx}{dt} = 3e^{\frac{t}{2}}$$

$$\Rightarrow \int dx = \int 3e^{\frac{t}{2}} dt$$

$$\Rightarrow x = 6e^{\frac{t}{2}} + D$$

In first 4 secs:

$$x|_4 = 6e^{\frac{4}{2}} + D$$

In first 5 secs:

$$x|_5 = 6e^{\frac{5}{2}} + D$$

In fifth second:

$$\text{dist} = x|_5 - x|_4 = 6[e^{\frac{5}{2}} - e^2]$$

D cancels.