1980 Q10:

$$\frac{d^2y}{dx^2} + \frac{2}{y^3} = 0.$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{2}{y^3} \quad *$$

Let $p = \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{dp}{dx}$

$$(*) \Rightarrow \frac{dp}{dx} = -\frac{2}{y^3} \quad (**)$$

But $\frac{dp}{dx} = \frac{dp}{dy}\frac{dy}{dx} = p\frac{dp}{dy}$.

so $(**) \Rightarrow p\frac{dp}{dy} = -\frac{2}{y^3}$

$$\Rightarrow \int p\,dp = -2\int \frac{dy}{y^3} + A$$

$$\Rightarrow \int p\,dp = -2\int y^{-3}dy + A$$

$$\Rightarrow \frac{p^2}{2} = -2\frac{y^{-2}}{-2} + A$$

$$\Rightarrow \frac{p^2}{2} = \frac{1}{y^2} + A.$$

// $p = \sqrt{2}$ when $y = 1$.

$$\Rightarrow \frac{(\sqrt{2})^2}{2} = \frac{1}{1} + A$$

$$\Rightarrow \frac{2}{2} = 1 + A$$

$$\Rightarrow 0 = A \; //$$

$$\Rightarrow \frac{p^2}{2} = \frac{1}{y^2}$$

$$\Rightarrow p^2 = \frac{2}{y^2}$$

$$\Rightarrow p = \pm\sqrt{\frac{2}{y^2}}$$

$$\Rightarrow p = \pm\frac{\sqrt{2}}{y}$$

// $p = +\sqrt{2}$ when $y = 1$ means we should take the + answer //

$$\Rightarrow p = \frac{\sqrt{2}}{y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{2}}{y}$$

$$\Rightarrow \int y\,dy = \sqrt{2}\int dx + B$$

$$\Rightarrow \frac{y^2}{2} = \sqrt{2}x + B$$

// $x = \sqrt{2}$ when $y = 1$.

$$\Rightarrow \frac{1^2}{2} = \sqrt{2}(\sqrt{2}) + B$$

$$\Rightarrow \frac{1}{2} = 2 + B$$

$$\Rightarrow -\frac{3}{2} = B \; //$$

$$\Rightarrow \frac{y^2}{2} = \sqrt{2}x - \frac{3}{2}$$

$$\Rightarrow y^2 = 2\sqrt{2}x - 3$$

$$\Rightarrow y = \sqrt{2\sqrt{2}x - 3}$$

[why do I only take the + answer]

Initially $v = 0, s = 0$

Signs: $\xrightarrow{+}$

Forces: $\longrightarrow F = m(5 - v^2)$

NII $\Rightarrow m\frac{d^2s}{dt^2} = m(5 - v^2)$

$$\Rightarrow \frac{d^2s}{dt^2} = 5 - v^2.$$

Trying to link v and s.

$$\Rightarrow v\frac{dv}{ds} = 5 - v^2$$

$$\Rightarrow v\,dv = (5 - v^2)\,ds \quad \text{[qed!]}$$

$$\Rightarrow \int \frac{v}{5 - v^2}\,dv = \int ds + A$$

$\Rightarrow$ // Let

$u = 5 - v^2$

$\Rightarrow \frac{du}{dv} = -2v$

$\Rightarrow -\frac{1}{2}du = v\,dv$ //

$$\Rightarrow \int \frac{-\frac{1}{2}du}{u} = s + A$$

$$\Rightarrow -\frac{1}{2}\ln u = s + A$$

$$\Rightarrow -\frac{1}{2}\ln(5 - v^2) = s + A.$$

$$\Rightarrow \ln(5 - v^2)^{-\frac{1}{2}} = s + A.$$

// $s = 0$ when $v = 0$ //

$$\Rightarrow \ln(5)^{-\frac{1}{2}} = A$$

$$\Rightarrow \ln(5 - v^2)^{-\frac{1}{2}} = s + \ln 5^{-\frac{1}{2}}$$

$v = ?$ when $s = 1.5$.

[See BOARD]