

1979 100s

soln $x \frac{dy}{dx} = -y$

$\Rightarrow \int \frac{dy}{y} = \int -\frac{dx}{x}$

$\Rightarrow \ln y = -\ln x + C$

soln $x \frac{d^2 y}{dx^2} = -\frac{dy}{dx}$ (F)

Let $p = \frac{dy}{dx}$

$\Rightarrow \frac{d^2 y}{dx^2} = \frac{dp}{dx} = \frac{dp}{dx}$

(F) $\Rightarrow x \frac{dp}{dx} = -p$

$\Rightarrow \int \frac{dp}{p} = -\int \frac{dx}{x} + A$

$\Rightarrow \ln p = -\ln x + A$

Can't use given condition to find A yet.

Tidy up.

$\ln p + \ln x = A$

$\Rightarrow \ln(px) = A$

$\Rightarrow px = e^A$

Call $e^A = C$

$\Rightarrow px = C$

$\Rightarrow p = \frac{C}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{C}{x}$

$\Rightarrow \int dy = \int \frac{C}{x} dx + B$

$\Rightarrow y = \int \frac{dx}{x} + B$

$\Rightarrow y = \ln x + B$

Now use conditions to find C and B.

$y=0, x=1 \Rightarrow 0 = \ln(1) + B$

$\Rightarrow 0 = C + B$

$\Rightarrow 0 = B$

$y=3, x=e \Rightarrow 3 = \ln e$

$\Rightarrow 3 = C(1)$

$\Rightarrow C=3$

$\therefore y = 3 \ln x$

(B)

$a = -\frac{v^2}{10}$



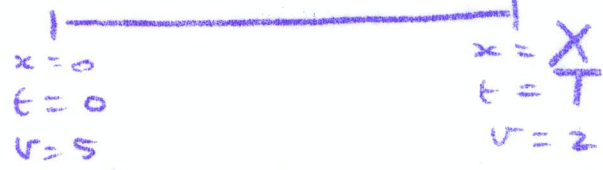
NII $\Rightarrow ma = \Sigma F$

$\Rightarrow ma = -\frac{mv^2}{10}$

$\Rightarrow a = -\frac{v^2}{10}$

Equation of motion

Position:



How long does it take for this drop in speed

Link v and t.

$a = -\frac{v^2}{10}$

$\Rightarrow \frac{dv}{dt} = -\frac{v^2}{10}$

$\Rightarrow \int \frac{dv}{v^2} = \int -\frac{1}{10} dt$

$\Rightarrow \left. -\frac{1}{v} \right|_5 = \left. -\frac{t}{10} \right|_0$

$\Rightarrow -\frac{1}{2} - \left(-\frac{1}{5}\right) = -\frac{T}{10}$

$\Rightarrow -\frac{1}{2} + \frac{1}{5} = -\frac{T}{10}$

$\Rightarrow -\frac{3}{10} = -\frac{T}{10}$

$\Rightarrow 3 = T$

Time is 3 secs.

How far does it go for this drop in speed.

Link v and x

$a = -\frac{v^2}{10}$

$\Rightarrow v \frac{dv}{dx} = -\frac{v^2}{10}$

$\Rightarrow \int \frac{2v}{v^2} dv = \int -\frac{x}{10} dx$

$\Rightarrow \int \frac{1}{v} dv = \int -\frac{x}{10} dx$

$\ln v \Big|_5 = -\frac{x}{10} \Big|_0$

$\ln 2 - \ln 5 = -\frac{x}{10}$

$\ln\left(\frac{2}{5}\right) = -\frac{x}{10}$

$10 \ln 0.4 = -x$

$-9.16 = -x$

9.16 metres = x