

1978 Q10 (BOUNDARY OF INTEGRATION)

(i) $\frac{dy}{dx} = \sqrt{1-y^2}, y=0, x=1$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{1}$

$\Rightarrow \sin^{-1} \frac{y}{1} = x + A$

$\Rightarrow \sin^{-1} 0 = 1 + A$

$\Rightarrow \sin^{-1} y = x - 1$

$\Rightarrow y = \sin(x-1)$

(ii) $\frac{d^2y}{dx^2} = -\frac{1}{y^3}, \frac{dy}{dx} = 1, x = \frac{1}{2}$
 both when $y=1$.

$\Rightarrow \frac{dp}{dx} = -\frac{1}{y^3}$ $p \equiv \frac{dp}{dx}$

$\Rightarrow p \frac{dp}{dy} = -\frac{1}{y^3}$

$\Rightarrow \int p dp = \int -\frac{1}{y^3} dy$

$\Rightarrow \frac{p^2}{2} = \frac{1}{2y^2} + C$

$\Rightarrow \frac{dy}{dx} = p = 1$ when $y=1$

$\Rightarrow \frac{1}{2} = \frac{1}{2} + C$

$\Rightarrow C = 0$

$\Rightarrow p^2 = \frac{1}{y^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$ (why take + sign?)

$\Rightarrow \int y dy = \int dx + D$

$\Rightarrow \frac{y^2}{2} = x + D$

$\Rightarrow x = \frac{1}{2}, y = 1 \Rightarrow$

$\frac{1}{2} = \frac{1}{2} + D$

$\Rightarrow D = 0$

$\Rightarrow y^2 = 2x \Rightarrow y = \sqrt{2x}$

(iii) + positive

0 distance d
 position $\leftarrow \rightarrow$

P (starts here)
 $v=0$

Force: $F = \frac{2m}{x^5}$

Particle starts from P.
 Distance $|op| = d$

NII $\Rightarrow \frac{2m}{x^5} = m a$

$\Rightarrow \frac{2}{x^5} = \frac{dv}{dt}$

OR $\frac{2}{x^5} = v \frac{dv}{dx}$ (E)

Boundary of Integration.

$x=d$ let $x=S$
 $v=0$ $v=V$

(q at typical point)

(E) $\Rightarrow \int_d^S \frac{2}{x^2} dx = \int_0^V v dv$

$\Rightarrow \left. -\frac{1}{2x^2} \right|_d^S = \left. \frac{v^2}{2} \right|_0^V$

$\Rightarrow \frac{-1}{S^2} + \frac{1}{d^2} = \frac{V^2}{2}$

$\Rightarrow V = \sqrt{\frac{1}{d^2} - \frac{1}{S^2}}$

speed at a typical distance, from the origin.

As $S \rightarrow \infty \frac{1}{S^2} \rightarrow 0$

$\Rightarrow V \rightarrow \sqrt{\frac{1}{d^2}} = \frac{1}{d}$

$V \rightarrow \frac{1}{d}$ qed.