AE 1976 Q4 : Origin O. SIGNS $\xrightarrow{+}$ (x measured from O.)

POSITIONS :

point o	point n	point p	point a
	$x = X$	$x = \frac{d}{2}$	$x = d$
	$v = 0$	$v = V$	$v = -\frac{2k\sqrt{3}}{d^2}$

FORCES :

$+mk^2x^{-5}$

N II $\Rightarrow$ $F = ma$ $\Rightarrow$ $mk^2x^{-5} = ma$

$\Rightarrow \boxed{\frac{k^2}{x^5} = \text{accel.}}$

Here linking v and x always $\Rightarrow \boxed{\frac{k^2}{x^5} = v\frac{dv}{dx}}$ ⊛

(i) Find $v = V$ at the point p

$$\int_d^{\frac{d}{2}} \frac{k^2}{x^5}\,dx = \int_{-\frac{2k\sqrt{3}}{d^2}}^{V} v\,dv$$

$$\Rightarrow -\frac{k^2}{4}x^{-4}\Big|_d^{\frac{d}{2}} = \frac{v^2}{2}\Big|_{-\frac{2k\sqrt{3}}{d^2}}^{V}$$

$$\Rightarrow -\frac{k^2}{2}\left[\left(\frac{d}{2}\right)^{-4} - (d)^{-4}\right] = v^2\Big|_{-\frac{2k\sqrt{3}}{d^2}}^{V}$$

$$\Rightarrow -\frac{k^2}{2}\left[\frac{2^4}{d^4} - \frac{1}{d^4}\right] = V^2 - \left(\frac{-2k\sqrt{3}}{d^2}\right)^2$$

$$\Rightarrow -\frac{k^2}{2}\left[\frac{16-1}{d^4}\right] = V^2 - \frac{4k^2 3}{d^4}$$

$$\Rightarrow -\frac{15k^2}{2d^4} = V^2 - \frac{12k^2}{d^4}.$$

$$\Rightarrow -\frac{15k^2}{2d^4} + \frac{12k^2}{d^4} = V^2$$

$$\Rightarrow \frac{k^2}{d^4}\left(\frac{-15+24}{2}\right) = V^2$$

$$\Rightarrow \frac{k^2}{d^4}\left(\frac{9}{2}\right) = V^2$$

$$\Rightarrow \boxed{\frac{k}{d^2}\frac{3}{\sqrt{2}} = V}$$

is speed at p.

(ii) Find $x = ?$ where comes to rest ie. where $v = 0$. [At point n]

$\int dx$ ⊛ $\Rightarrow$ (as before)

$$-\frac{k^2}{4}x^{-4}\Big| = \frac{v^2}{2}\Big|$$

Here

$$-\frac{k^2}{4}x^{-4}\Big|_d^{X} = \frac{v^2}{2}\Big|_{-\frac{2k\sqrt{3}}{d^2}}^{0}$$

$$\Rightarrow -\frac{k^2}{2}\left[X^{-4} - d^{-4}\right] = 0^2 - \left(\frac{-2k\sqrt{3}}{d^2}\right)^2$$

$$\Rightarrow -\frac{k^2}{2}\left[\frac{1}{X^4} - \frac{1}{d^4}\right] = -\frac{12k^2}{d^4}.$$

$$\Rightarrow \frac{1}{X^4} - \frac{1}{d^4} = \frac{24}{d^4}$$

$$\Rightarrow \frac{1}{X^4} = \frac{24}{d^4} + \frac{1}{d^4}$$

$$\Rightarrow \frac{1}{X^4} = \frac{25}{d^4}$$

$$\Rightarrow X^4 = \frac{d^4}{25}$$

$$\Rightarrow X = \frac{d}{\sqrt{5}}.\ \text{metre}$$

So particles reaches point n which is $\frac{d}{\sqrt{5}}$ m from O.