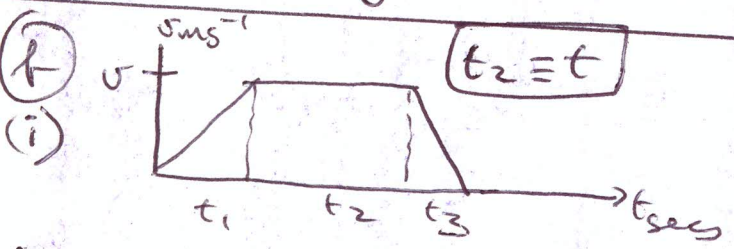


$h = ut + \frac{1}{2}gt^2$
 $\Rightarrow 2h = 2ut - gt^2$
 $\Rightarrow gt^2 - 2ut + 2h = 0$

$\Rightarrow t^2 - \frac{2u}{g}t + \frac{2h}{g} = 0$

t_1, t_2 are roots of this quadratic
 $\Rightarrow t_1, t_2 = \text{independent terms}$

$\Rightarrow t_1 \cdot t_2 = \frac{2h}{g}$



Average speed = $\frac{3v}{4}$

$\Rightarrow \frac{\text{Total Dist}}{\text{Total time}} = \frac{3v}{4}$ (1)

Area under curve = distance travelled.

$\Rightarrow \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3 = \text{Total distance}$ (2)

(1) and (2) \Rightarrow

$\frac{\frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3}{t_1 + t_2 + t_3} = \frac{3v}{4}$

$\Rightarrow \frac{1}{2}t_1 + t_2 + \frac{1}{2}t_3 = \frac{3}{4}(t_1 + t_2 + t_3)$

$\frac{1}{2}t_1 + t_2 + \frac{1}{2}t_3 = \frac{3}{4}t_1 + \frac{3}{4}t_2 + \frac{3}{4}t_3$

$2t_1 + 4t_2 + 2t_3 = 3t_1 + 3t_2 + 3t_3$

$2t_1 + 2t_2 + 2t_3 + 2t_2 = 3(t_1 + t_2 + t_3)$

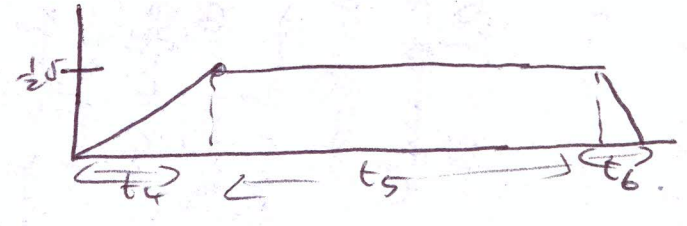
$2(T) + 2t_2 = 3T$

$\Rightarrow 2t_2 = 3T - 2T$

$\Rightarrow 2t_2 = T$

$\Rightarrow \boxed{2t_2 = T}$ q.e.d. (10)

(ii) Speed limit $\frac{1}{2}v$
Total distance still is in (i).



From (i):

$\frac{\text{Total dist}}{\text{Total time}} = \frac{3v}{4}$

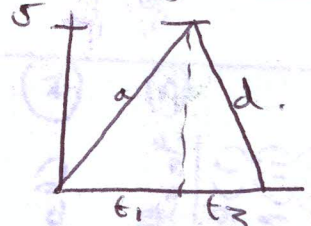
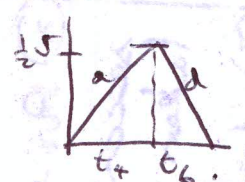
$\Rightarrow \text{Total dist} = \frac{3v}{4} (\text{Total time})$

$\Rightarrow \text{Total dist} = \frac{3v}{4} (2t)$

$\Rightarrow \text{Total dist} = \frac{3vt}{2}$ (5)

Examine accel/decel regions.

New.



As accel/decel same in both situation

then $(t_4 + t_6) : (t_1 + t_3)$ as $\frac{1}{2}v : v$.

$\Rightarrow t_4 + t_6 = \frac{1}{2}(t_1 + t_3)$

But $t_1 + t_3 = T - t = 2t - t = t$

$\therefore t_4 + t_6 = \frac{1}{2}t$ (5)

Find t_5 : Area under curve = dist travelled.

$\Rightarrow \frac{1}{2}(\frac{1}{2}v)t_4 + \frac{1}{2}(\frac{1}{2}v)t_6 + \frac{1}{2}vt_5 = \frac{3v}{4}t$

$\Rightarrow \frac{1}{2}(t_4 + t_6) + t_5 = 3t$

$\Rightarrow \frac{1}{2}(\frac{1}{2}t) + t_5 = 3t$

$\Rightarrow t_5 = 3t - \frac{1}{4}t = \frac{11}{4}t$

\therefore Total new time:

$t_4 + t_5 + t_6 = t_4 + t_6 + t_5$

$= \frac{1}{2}t + \frac{11}{4}t$

$= \frac{13}{4}t \text{ sec}$ (5)