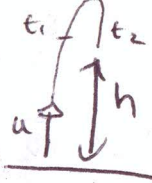


$h = ut - \frac{1}{2}gt^2$

$\Rightarrow 2h = 2ut - gt^2$

$\Rightarrow gt^2 - 2ut + 2h = 0$

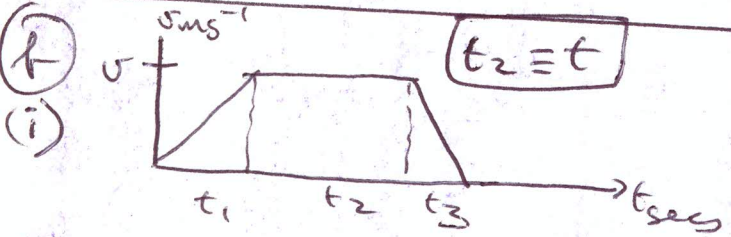
$\Rightarrow t^2 - \frac{2u}{g}t + \frac{2h}{g} = 0.$

t_1, t_2 are roots of this quadratic

$\Rightarrow t_1 . t_2$ = independent term.

$\Rightarrow t_1 . t_2 = \frac{2h}{g}$

(i)



$t_2 \equiv t$

Average speed $= \frac{3v}{4}$

$\Rightarrow \frac{\text{Total Dist}}{\text{Total time}} = \frac{3v}{4}$ (1)

Area under curve = distance travelled.

$\Rightarrow \frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3 = \text{Total distance}$ (2)

(1) and (2) $\Rightarrow$

$$\frac{\frac{1}{2}vt_1 + vt_2 + \frac{1}{2}vt_3}{t_1 + t_2 + t_3} = \frac{3v}{4}$$

$\Rightarrow \frac{1}{2}t_1 + t_2 + \frac{1}{2}t_3 = \frac{3}{4}(t_1 + t_2 + t_3)$

$\frac{1}{2}t_1 + t_2 + \frac{1}{2}t_3 = \frac{3}{4}t_1 + \frac{3}{4}t_2 + \frac{3}{4}t_3$

$2t_1 + 4t_2 + 2t_3 = 3t_1 + 3t_2 + 3t_2$

$2t_1 + 2t_2 + 2t_3 + 2t_2 = 3(t_1 + t_2 + t_3)$

$\Rightarrow 2(T) + 2t_2 = 3T$

$\Rightarrow 2t_2 = 3T - 2T$

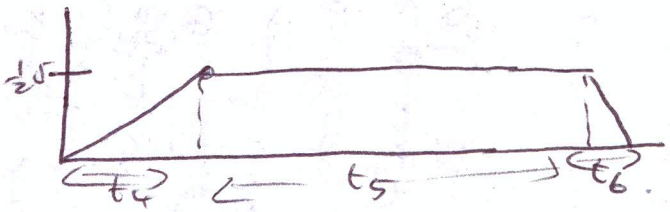
$\Rightarrow 2t_2 = T$

$\Rightarrow \boxed{2t = T}$ (10)

qed.

(ii) Speed limit $\frac{1}{2}v$

Total distance still v in (i).



From B(i):

$\frac{\text{Total dist}}{\text{Total time}} = \frac{3v}{4}$.

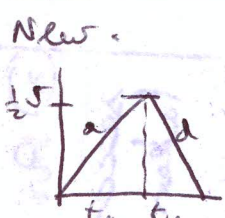
$\Rightarrow$ Total dist $= \frac{3v}{4}$ (Total time)

$\Rightarrow$ Total dist $= \frac{3v}{4}(2t)$.

$\Rightarrow \boxed{\text{Total dist} = \frac{3vt}{2}}$ (5)

Examine accel/decel regions.

New.



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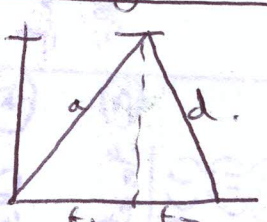

As accel/decel same in both situations

then $(t_4 + t_6) : (t_1 + t_3)$ as $\frac{1}{2}v : v$.

$\Rightarrow t_4 + t_6 = \frac{1}{2}(t_1 + t_3)$

But $t_1 + t_3 = T - t = 2t - t = t$.

$\therefore t_4 + t_3 = \frac{1}{2}t$. (5)

Find t_5: Area under curve = dist Travelled

$\Rightarrow \frac{1}{2}\left(\frac{1}{2}v\right)t_4 + \frac{1}{2}\left(\frac{1}{2}v\right)t_6 + \frac{1}{2}vt_5 = \frac{3v}{2}t$.

$\Rightarrow \frac{1}{2}(t_4 + t_6) + t_5 = 3t$

$\Rightarrow \frac{1}{2}\left(\frac{1}{2}t\right) + t_5 = 3t$

$\Rightarrow t_5 = 3t - \frac{1}{4}t = \frac{11}{4}t$.

$\therefore$ Total new time:

$t \quad t_4 + t_5 + t_6 = t_4 + t_6 + t_5$

$= \frac{1}{2}t + \frac{11}{4}t$

$= \frac{13}{4}t$ secs (5)